Hydraulics

Solution Sheet 9 – Basic hydraulics of flow in pipes

1. *A horizontal* $50\,\mathrm{mm}$ *pipeline leaves a water tank with a square-edged entrance at* $H = 6\,\mathrm{m}$ *below the water surface and discharges into the atmosphere. Calculate the flow rate, assuming a friction factor* $\lambda = 0.025$ *and* $\alpha = 1.2$ *for four cases, with and without entrance loss* $K = 0.5$*, and for a pipe length* L *of* $4.5\,\mathrm{m}$ *and* $45\,\mathrm{m}$.

   We apply the energy equation between the surface of the reservoir and the outlet of the pipe, where pressure is atmospheric in both cases:

$$
\begin{aligned}
\left(\frac{p}{\rho g} + z + \frac{\alpha}{2g}\frac{Q^2}{A^2}\right)_{\text{in}} &= \left(\frac{p}{\rho g} + z + \frac{\alpha}{2g}\frac{Q^2}{A^2}\right)_{\text{out}} + \Delta H \\
0 + H + 0 &= 0 + 0 + \frac{\alpha}{2g}\frac{Q^2}{A^2} + \frac{K}{2g}\frac{Q^2}{A^2} + \lambda\frac{L}{D}\frac{1}{2g}\frac{Q^2}{A^2}, \quad \text{giving} \\
Q &= A\sqrt{\frac{2gH}{\alpha + K + \lambda L/D}} \\
&= \frac{\pi}{4}0.05^2\sqrt{\frac{12\times 9.8}{1.2 + K + 0.025\,L/0.05}}.
\end{aligned}
$$

   Notice that the velocity distribution coefficient α in the kinetic head term appears as another local loss. Evaluating this expression:

   a. $K = 0.5$, $L = 4.5\,\mathrm{m}$, $Q = 0.0107\,\mathrm{m}^3\,\mathrm{s}^{-1}$.

   b. $K = 0$, $L = 4.5\,\mathrm{m}$, $Q = 0.0115\,\mathrm{m}^3\,\mathrm{s}^{-1}$, hence for a short pipe, ignoring local loss changed results by some 8%.

   c. $K = 0.5, L = 45\,\mathrm{m}, Q = 0.00433\,\mathrm{m}^3\,\mathrm{s}^{-1}$, so that a 10-fold increase in pipe length has reduced discharge by a factor of (roughly) $\sqrt{10} \approx 3$.

   d. $K = 0$, $L = 45\,\mathrm{m}, Q = 0.00437\,\mathrm{m}^3\,\mathrm{s}^{-1}$, and now, for this long pipe, neglecting K has had little effect.

2. *Now generalising, to estimate the relative importance of the velocity correction factor* α*, a local loss* K*, and friction* λ*: a horizontal pipeline of diameter* D *a depth* H *below the surface leaves a water tank with entrance loss coefficient* K *and discharges into the atmosphere. The pipe has length* L *and friction factor* λ*. Calculate the flow rate in terms of these quantities (you could use the solution from the previous question) and using the total differential explore the sensitivity of the discharge* Q *to* α*,* K*, and* λ *by considering realistic values and uncertainties in those quantities for pipeline length/diameter ratios of* 100 *and* 1000*. Consider* α *in the range* $1 - 1.3$*,* K *might be, say,* 0.5 ± 0.15*, and the value of* λ *from the Moody diagram might be* 0.02 ± 0.005*, corresponding to uncertainties of 100% in the relative roughness.*

   From above we write down

$$
Q = A\sqrt{\frac{2gH}{\alpha + K + \lambda L/D}}.
$$

   Now considering the total differential, to estimate the importance of varying each quantity:

$$
\begin{aligned}
dQ &= \frac{\partial Q}{\partial \alpha}d\gamma + \frac{\partial Q}{\partial K}dK + \frac{\partial Q}{\partial \lambda}d\lambda \\
\frac{\partial Q}{\partial \alpha} &= -\tfrac{1}{2}\frac{A\sqrt{2gH}}{(\alpha + K + \lambda L/D)^{3/2}}, \text{ etc.,}
\end{aligned}
$$

Dividing by Q to get the fractional change gives

$$\frac{dQ}{Q} = -\tfrac{1}{2}\frac{d\gamma + dK + L/D \times d\lambda}{\alpha + K + \lambda\frac{L}{D}},$$

and we see that the relative effects are simply in the ratio of $d\gamma : dK : L/D \times d\lambda$. The value of α could be as much as, say, 1.3 compared with the traditional assumption that $\alpha = 1$; the value of K might be, say, 0.5 ± 0.15; and the value of λ from the Moody diagram might be 0.02 ± 0.005, corresponding to uncertainties of 100% in the relative roughness. These give us the relative effects on discharge as

$$\begin{aligned}
\text{Coefficient } \alpha &: \quad \frac{dQ}{Q} = -\tfrac{1}{2}\frac{d\gamma}{\alpha + K + \lambda\frac{L}{D}} = -\tfrac{1}{2}\frac{0.3}{1 + 0.5 + 0.02 \times 100} \approx -4\% \\
\text{Coefficient } K &: \quad \frac{dQ}{Q} = -\tfrac{1}{2}\frac{dK}{\alpha + K + \lambda\frac{L}{D}} = -\tfrac{1}{2}\frac{0.3}{1 + 0.5 + 0.02 \times 100} \approx -4\% \\
\text{Coefficient } \lambda &: \quad \frac{dQ}{Q} = -\tfrac{1}{2}\frac{L/D \times d\lambda}{\alpha + K + \lambda\frac{L}{D}} = -\tfrac{1}{2}\frac{100 \times 0.01}{1 + 0.5 + 0.02 \times 100} \approx -14\%
\end{aligned}$$

Repeating these calculations for $L/D = 1000$ gives -1%, -1%, -23% respectively. It seems that for pipes of a realistic length, knowledge of the friction factor is more important for accuracy.

3. *Do Example 16 on page 74 of the lecture notes for yourself, paying attention to the plotting of the piezometric head line and the total head line. It might be better to do all calculations and plotting using a spreadsheet.*

4. *Repeat, but where instead of a nozzle, the pipe discharges into a tank where the water surface is* $1\,\mathrm{m}$ *above the pipe. Include the exit loss in your calculations, and again, carefully plot the lines mentioned in Question 1.*

   Consider the pipeline emerging from a tank, passing through a valve, and finally the flow emerges into a tank where the surface is $1\,\mathrm{m}$ above the pipe, as shown in the figure. Determine the elevation of hydraulic and energy grade lines at points A-E. Use $\alpha = 1.3$.First we apply the integral energy

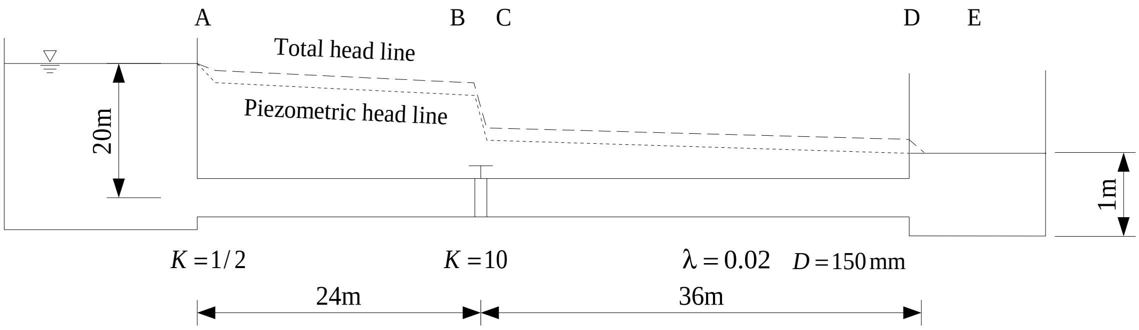


Figure 1.

equation between the surfaces of the upper tank and of the lower tank:

$$\left(\frac{p}{\rho g} + z + \frac{\alpha}{2g}\frac{Q^2}{A^2}\right)_{\text{in}} = \left(\frac{p}{\rho g} + z + \frac{\alpha}{2g}\frac{Q^2}{A^2}\right)_{\text{out}} + \Delta H$$

which gives (where the exit loss is now the whole kinetic head)

$$0 + 20 + 0 = 0 + 1 + 0 + \frac{Q^2}{2g\left(\pi D^2/4\right)^2}\left(\underbrace{\frac{1}{2}}_{K_A} + \underbrace{10}_{K_{BC}} + \underbrace{\frac{0.02 \times 60}{0.15}}_{\text{Friction: } \lambda L/D} + \underbrace{\alpha}_{\text{Exit loss}}\right)$$

$$19 = \frac{Q^2}{2 \times 9.8 \times \left(\pi 0.15^2/4\right)^2} \left(\tfrac{1}{2} + 10 + 8 + 1.3\right)$$

We obtain a discharge of $Q = 0.0766\,\mathrm{m}^3\,\mathrm{s}^{-1}$. Hence, our basic term

$$\frac{U^2}{2g} = \frac{Q^2}{2g\left(\pi 0.15^2/4\right)^2} = \frac{0.0766^2}{2 \times 9.8 \times \left(\pi 0.15^2/4\right)^2} = 0.959\,\mathrm{m},$$

to be used in local loss formulae.

Now we apply the energy equation between the surface of the tank and just after A, and then between each of the subsequent points:

$H_A = 20 - 0.5 \times 0.959 = 19.52$ $\qquad h_A = 19.52 - 1.3 \times 0.96 = 18.27$

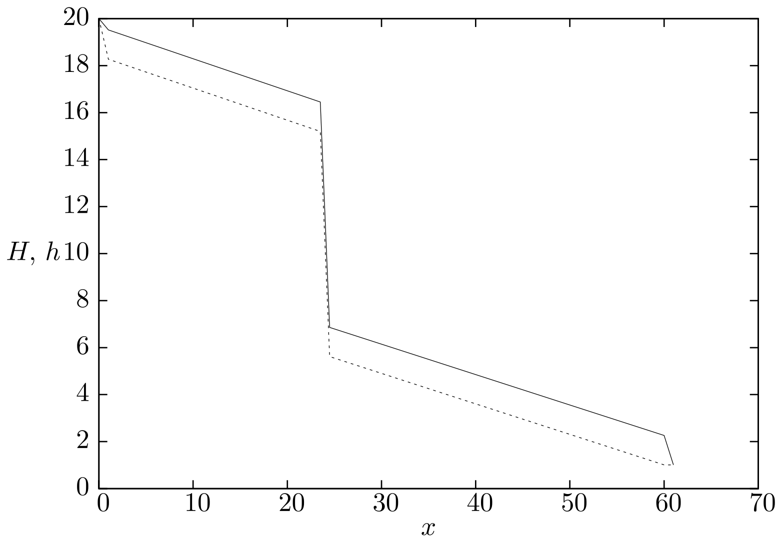
$H_B = 19.52 - \frac{0.02 \times 24}{0.15} \times 0.959 = 16.45$ $\qquad h_B = 16.45 - 1.3 \times 0.96 = 15.20$

$H_C = 16.45 - 10 \times 0.959 = 6.86$ $\qquad h_C = 6.86 - 1.3 \times 0.96 = 5.62$

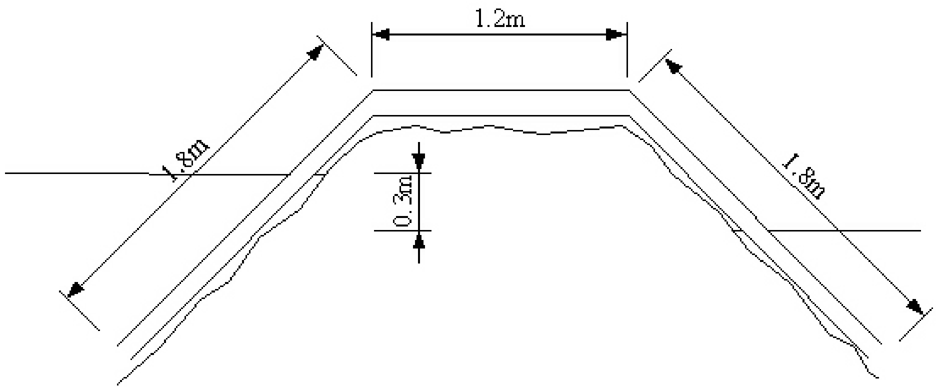
$H_D = 6.86 - \frac{0.02 \times 36}{0.15} \times 0.959 = 2.26$ $\qquad h_D = 2.26 - 1.3 \times 0.96 = 1.01$

$H_E = 2.26 - 1.3 \times 0.959 = 1.01$ $\qquad h_E = 1.01$

The final value of the piezometric head and total head at E should have been 1.00, but accumulated round-off error by working to two places only, has caused an error of $1\,\mathrm{cm}$. These results are plotted in the figure.



5. *An irrigation siphon placed over a canal bank is a pipe with a diameter of* $100\,\mathrm{mm}$ *and the dimensions shown. Estimate the flowrate for a head of* $0.3\,\mathrm{m}$. *Assume a re-entrant loss of* $K = 0.8$*; the full kinetic energy loss at the exit, but where* $\alpha = 1$ *because the flow distribution in the pipe has not had time to develop; a friction factor of* $\lambda = 0.02$ *and bend loss coefficients of* $K = 0.2$.



Applying the energy conservation law between the surface on one side and the surface on the other:

$$\begin{aligned}
\left(\frac{p}{\rho g}+z+\frac{\alpha}{2g}\frac{Q^2}{A^2}\right)_{\text{in}} &= \left(\frac{p}{\rho g}+z+\frac{\alpha}{2g}\frac{Q^2}{A^2}\right)_{\text{out}}+\Delta H \\
0+0.3+0 &= 0+0+0+\frac{K_{\text{entry}}}{2g}\frac{Q^2}{A^2}+2\times\frac{K_{\text{bends}}}{2g}\frac{Q^2}{A^2}+\frac{\alpha}{2g}\frac{Q^2}{A^2}+\lambda\frac{L}{D}\frac{1}{2g}\frac{Q^2}{A^2} \\
Q &= A\sqrt{\frac{2g\times 0.3}{K_{\text{entry}}+2\times K_{\text{bends}}+\alpha+\lambda L/D}} \\
&= \frac{\pi}{4}\times 0.1^2\sqrt{\frac{2\times 9.8\times 0.3}{0.8+2\times 0.2+1+0.02(1.8+1.2+1.8)/0.1}} \\
&= 0.0107\,\mathrm{m}^3\,\mathrm{s}^{-1}=10.7\,\mathrm{L}\,\mathrm{s}^{-1}
\end{aligned}$$

6. *A town water supply has a reservoir whose surface is $100\,\mathrm{m}$ higher than that in the water tower in the town which acts as a distribution reservoir, which stores enough to average out flows between times of high and low demand. This means that when we design the pipeline joining the two we can consider mean demand, which is $250L$ per head per day. The town is estimated to have a population of $4,000$ people at the end of the 20 year design life. The reservoir and the tower are $5\,\mathrm{km}$ apart so that we can ignore local losses. The pipes to be used are concrete, with an equivalent roughness of $1\,\mathrm{mm}$. Design the pipe – that is, determine its minimum diameter so that the mean flow can be carried. Of course, in practice, the next larger commercial pipe size would be used.*

   Considering equation (9.15) in the lecture notes which is a nonlinear transcendental equation, written in a form suitable for an iterative solution:

$$D=\left(-\frac{Q}{\pi}\sqrt{\frac{2}{i\,g}}\log_{10}^{-1}\left(\frac{d/D}{3.7}+\frac{2.51\,\nu}{D^{3/2}\sqrt{2\,i\,g}}\right)\right)^{2/5}.$$

   An initial value of D is assumed, substituted on the right, and an updated value calculated. The other quantities are:

$$\begin{aligned}
Q &= 4000\times 250\text{ L/d}=\frac{4000\times 250}{24\times 3600\times 1000}=0.0116\,\mathrm{m}^3\,\mathrm{s}^{-1} \\
i &= \frac{\Delta H}{L}=\frac{100}{5\times 1000}=0.02 \\
d &= 0.001 \\
\nu &= 1\times 10^{-6}\,\mathrm{m}^2\,\mathrm{s}^{-1}
\end{aligned}$$

   The recursion expression is

$$D=\left(-\frac{0.0116}{\pi}\sqrt{\frac{2}{0.02\times 9.8}}\log_{10}^{-1}\left(\frac{0.001/D}{3.7}+\frac{2.51\times 10^{-6}}{D^{3/2}\sqrt{2\times 0.02\times 9.8}}\right)\right)^{2/5}.$$

   Let us assume an initial diameter $D=1\,\mathrm{m}$, and evaluating iteratively gives successively $D=0.102\,\mathrm{m}$, $D=0.116\,\mathrm{m}$, $D=0.115\,\mathrm{m}$, $D=0.115\,\mathrm{m}$. Hence the process has converged, with a required diameter of at least $0.115\,\mathrm{m}$. This would be easily done using a spreadsheet.

7. *Now generalising, to estimate the relative importance of the velocity correction factor α, a local loss K, and friction λ: a horizontal pipeline of diameter D a depth H below the surface leaves a water tank with entrance loss coefficient K and discharges into the atmosphere. The pipe has length L and friction factor λ. Calculate the flow rate in terms of these quantities (you could use the solution from the previous question) and using the total differential explore the sensitivity of the discharge Q to α, K, and λ by considering realistic values and uncertainties in those quantities for pipeline length/diameter ratios of 100 and 1000. Consider α in the range $1-1.3$, K might be, say, 0.5 ± 0.15, and the value of λ from the Moody diagram might be 0.02 ± 0.005, corresponding to uncertainties of 100% in the relative roughness.*

From above we write down

$$Q = A\sqrt{\frac{2gH}{\alpha + K + \lambda L/D}}.$$

Now considering the total differential, to estimate the importance of varying each quantity:

$$\begin{aligned} dQ &= \frac{\partial Q}{\partial \alpha}d\gamma + \frac{\partial Q}{\partial K}dK + \frac{\partial Q}{\partial \lambda}d\lambda \\ \frac{\partial Q}{\partial \alpha} &= -\tfrac{1}{2}\frac{A\sqrt{2gH}}{(\alpha + K + \lambda L/D)^{3/2}}, \text{ etc.,} \end{aligned}$$

Dividing by Q to get the fractional change gives

$$\frac{dQ}{Q} = -\tfrac{1}{2}\frac{d\gamma + dK + L/D \times d\lambda}{\alpha + K + \lambda \frac{L}{D}},$$

and we see that the relative effects are simply in the ratio of $d\gamma : dK : L/D \times d\lambda$. The value of α could be as much as, say, 1.3 compared with the traditional assumption that $\alpha = 1$; the value of K might be, say, 0.5 ± 0.15; and the value of λ from the Moody diagram might be 0.02 ± 0.005, corresponding to uncertainties of 100% in the relative roughness. These give us the relative effects on discharge as

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