Hydraulics

Solution Sheet 10 – Pipe networks

1. *Aviation fuel flows from tank A to tank B through two pipes connected in series. Determine the discharge, given the data:*

	Pipe 1	Pipe 2
Length L	$300\,\mathrm{m}$	$240\,\mathrm{m}$
Diameter D	$600\,\mathrm{mm}$	$1\,\mathrm{m}$
Roughness	$2\,\mathrm{mm}$	$0.3\,\mathrm{mm}$
Total head loss ΔH	$6\,\mathrm{m}$	
Kinematic viscosity ν	$3\times10^{-6}\,\mathrm{m^2\,s^{-1}}$	
K_{entry}	0.5	
$\Delta H_{\text{expansion}}$	$(U_1-U_2)^2/2g$	
$K_{\text{exit}}=\alpha$	1.3	

Use the Haaland approximation for the friction coefficient λ. (*Ans*: $0.79\,\mathrm{m^3\,s^{-1}}$).

From the lecture notes on pipes in series,

$$\Delta H=\frac{Q^2}{2g}\left(\frac{K_1}{A_1^2}+\frac{\lambda_2L_2/D_2}{A_2^2}+\ldots+\frac{\lambda_6L_6/D_6}{A_6^2}+\frac{\alpha}{A_7^2}\right)=\frac{Q^2}{2g}\sum_{i=1}^{N}\frac{K_i}{A_i^2},$$

so that in this case we have

$$\Delta H=\frac{Q^2}{2g}\left(\frac{K_{\text{entry}}}{A_1^2}+\frac{\lambda_1L_1/D_1}{A_1^2}+\left(\frac{1}{A_1}-\frac{1}{A_2}\right)^2+\frac{\lambda_2L_2/D_2}{A_2^2}+\frac{\alpha}{A_2^2}\right).$$

We have $A_1=\pi/4\times0.6^2=0.283\,\mathrm{m}^2$, $A_2=\pi/4\times1^2=0.785$, $\varepsilon_1=d_1/D_1=0.002/0.600=0.0033$, $\varepsilon_2=d_2/D_2=0.0003/1=0.0003$, such that

$$6=\frac{Q^2}{2\times9.8}\left(\frac{0.5}{0.283^2}+\frac{\lambda_1 300/0.6}{0.283^2}+\left(\frac{1}{0.283}-\frac{1}{0.785}\right)^2+\frac{\lambda_2 240/1}{0.785^2}+\frac{1.3}{0.785^2}\right)$$

giving

$$Q=\sqrt{\frac{117.6}{13.459+6243\,\lambda_1+389.5\,\lambda_2}}\tag{1}$$

The Haaland approximation is

$$\begin{aligned}\lambda_1&=\frac{1}{1.8^2}\log_{10}^{-2}\left(\left(\frac{\varepsilon_1}{3.7}\right)^{10/9}+\frac{6.9}{\mathbf{R}_1}\right),\ \text{where } \mathbf{R}_1=\frac{4Q_1}{\nu\pi D_1},\\&=\frac{1}{1.8^2}\log_{10}^{-2}\left(\left(\frac{\varepsilon_1}{3.7}\right)^{10/9}+\frac{6.9\nu\pi D_1}{4Q_1}\right)\\&=\frac{1}{1.8^2}\log_{10}^{-2}\left(\left(\frac{0.0033}{3.7}\right)^{10/9}+\frac{6.9\times3\times10^{-6}\pi0.6}{4Q}\right)\\&=\frac{1}{1.8^2}\log_{10}^{-2}\left(0.00041+\frac{9.7546\times10^{-6}}{Q}\right)\qquad(2)\end{aligned}$$

$$\begin{aligned}\lambda_2&=\frac{1}{1.8^2}\log_{10}^{-2}\left(\left(\frac{0.0003}{3.7}\right)^{10/9}+\frac{6.9\times3\times10^{-6}\pi\times1}{4Q}\right)\\&=\frac{1}{1.8^2}\log_{10}^{-2}\left(2.85\times10^{-5}+\frac{1.63\times10^{-5}}{Q}\right)\qquad(3)\end{aligned}$$

Now we evaluate the three numbered equations in turn as an iteration process. We begin with (2), initially ignoring the second term so that our first approximation is the fully rough, infinite Reynolds number approximation:

$$
\begin{aligned}
\lambda_1 &= \frac{1}{1.8^2}\,\log_{10}^{-2}(0.00041) = 0.027 \\
\lambda_2 &= \frac{1}{1.8^2}\,\log_{10}^{-2}\left(2.85\times 10^{-5}\right) = 0.015 \\
Q &= \sqrt{\frac{117.6}{13.459 + 6243\ \times 0.027 + 389.5\ \times 0.015}} = 0.79\,\mathrm{m^3\,s^{-1}} \\
\lambda_1 &= \frac{1}{1.8^2}\,\log_{10}^{-2}\left(0.00041 + \frac{9.75\times 10^{-6}}{0.79}\right) = 0.027 \text{ (no change!)} \\
\lambda_2 &= \frac{1}{1.8^2}\,\log_{10}^{-2}\left(2.85\times 10^{-5} + \frac{1.63\times 10^{-5}}{0.79}\right) = 0.017 \\
Q &= \sqrt{\frac{117.6}{13.459 + 6243\ \times 0.027 + 389.5\ \times 0.017}} = 0.79\,\mathrm{m^3\,s^{-1}},
\end{aligned}
$$

and it can be seen that the process has converged. The discharge is $0.79\,\mathrm{m^3\,s^{-1}}$.

2. *A new reservoir supplies water to a service reservoir* $10\,\mathrm{km}$ *away with a total supply head of* $100\,\mathrm{m}$. *It is required to supply* $200\,\mathrm{L\,s^{-1}}$, *at* $20\,^\circ\mathrm{C}$ *(*$\nu = 10^{-6}\,\mathrm{m^2\,s^{-1}}$*). Initially allow for an entry loss of* $K = 0.5$ *and an exit loss of* $\alpha = 1.3$. *Use the Haaland approximation for* λ.

   a. *Justify your decision to ignore entry and exit losses.*

   b. *Design the pipeline, namely calculate the diameter necessary, if it is to be made of galvanised iron with a roughness of* $d = 0.03\,\mathrm{mm}$, *and round up to the nearest multiple of* $25\,\mathrm{mm}$.

   c. *What is the unregulated discharge in the pipeline?*

   d. *Calculate the head loss to be provided by a valve to regulate the flow to* $200\,L\,\mathrm{s^{-1}}$.

   (*Ans*: $350\,\mathrm{mm}$, $217\,\mathrm{L\,s^{-1}}$, $15\,\mathrm{m}$).

   (a) We have

$$
\begin{aligned}
\Delta H &= \frac{Q^2}{2gA^2}\left(K_{\text{entry}} + \frac{\lambda L}{D} + \alpha\right) \\
100 &= \frac{0.2^2}{2\times 9.8\times(\pi/4\times D^2)^2}\left(0.5 + \frac{\lambda\times 10000}{D} + 1.3\right).
\end{aligned}
$$

We might expect λ to be something like 0.03 and D to be something like $0.2\,\mathrm{m}$. Hence the frictional term will be approximately $0.03\times 10000/0.2 = 1500$, hence in comparison with 0.5 and 1.3 this is completely dominant.

(b) The Weisbach equation then becomes

$$100 = \frac{0.2^2}{2\times 9.8\times(\pi/4\times D^2)^2}\left(\frac{\lambda\times 10000}{D}\right),$$

and although λ is a function of D, it is not such a strongly varying one, and so we re-arrange this to give an expression for D:

$$D = (0.331\,\lambda)^{1/5}\,, \tag{4}$$

which we will use as part of an iteration scheme. The other part is the Haaland formula:

$$\lambda = \frac{1}{1.8^2}\,\log_{10}^{-2}\left(\left(\frac{\varepsilon}{3.7}\right)^{10/9} + \frac{6.9\nu\pi D}{4\,|Q|}\right)$$

$$\begin{aligned}
\lambda &= \frac{1}{1.8^2}\log_{10}^{-2}\left(\left(\frac{\varepsilon}{3.7}\right)^{10/9}+\frac{6.9\nu\pi D}{4Q}\right)\\
&= \frac{1}{1.8^2}\log_{10}^{-2}\left(\left(\frac{0.00003/D}{3.7}\right)^{10/9}+\frac{6.9\times10^{-6}\pi D}{4\times 0.2}\right)\\
&= \frac{1}{1.8^2}\log_{10}^{-2}\left(2.20\times10^{-6}\times D^{-10/9}+2.71\times10^{-5}D\right). \qquad (5)
\end{aligned}$$

Now we iterate on equations (4) and (5), assuming an initial $\lambda = 0.03$:

$$\begin{aligned}
D &= (0.331\times 0.03)^{1/5} = 0.400\\
\lambda &= \frac{1}{1.8^2}\log_{10}^{-2}\left(2.20\times10^{-6}\times 0.4^{-10/9}+2.71\times10^{-5}\times 0.4\right) = 0.0136\\
D &= (0.331\times 0.0136)^{1/5} = 0.339\\
\lambda &= \frac{1}{1.8^2}\log_{10}^{-2}\left(2.20\times10^{-6}\times 0.339^{-10/9}+2.71\times10^{-5}\times 0.339\right) = 0.0135\\
D &= (0.331\times 0.0135)^{1/5} = 0.339
\end{aligned}$$

Thus the iteration converged quickly, and we require a diameter of $339\,\mathrm{mm}$, so that the next largest available size is $350\,\mathrm{mm}$.

(c) In this case the discharge is

$$100 = \frac{Q^2}{2\times 9.8\times(\pi/4\times 0.35^2)^2}\left(\frac{0.0135\times 10000}{0.35}\right)$$

which gives $Q = 0.217\,\mathrm{m}^3\,\mathrm{s}^{-1}$.

(d) Head loss if flow is $200\,\mathrm{L}\,\mathrm{s}^{-1}$.

$$\Delta H = \frac{0.2^2}{2\times 9.8\times(\pi/4\times 0.35^2)^2}\left(\frac{0.0135\times 10000}{0.35}\right) = 85\,\mathrm{m},$$

hence the valve must provide $100 - 85 = 15\,\mathrm{m}$ head loss.

3. *(A worked version of this problem was included in the lecture notes – here it is modified slightly to include automatic allowance for flow direction).*

Three reservoirs are connected by pipelines which meet at a junction. The data for the reservoirs and corresponding pipelines are

Reservoir/pipeline	1	2	3
Surface elevation H	$30\,\mathrm{m}$	$18\,\mathrm{m}$	$9\,\mathrm{m}$
Pipe diameter D	$1\,\mathrm{m}$	$0.45\,\mathrm{m}$	$0.6\,\mathrm{m}$
Area $= \pi D^2/4$	$0.785\,\mathrm{m}^2$	$0.159\,\mathrm{m}^2$	$0.283\,\mathrm{m}^2$
Relative roughness d/D	0.0002	0.002	0.001
Pipeline length L	$3000\,\mathrm{m}$	$600\,\mathrm{m}$	$1000\,\mathrm{m}$

Calculate the discharge in each of the three pipes, assuming a viscosity of $\nu = 10^{-6}\,\mathrm{m}^2\,\mathrm{s}^{-1}$, and neglecting all local losses.

Use formulae given in the lecture notes which automatically allow for changes in flow direction:

$$Q_{iJ} = A\,\mathrm{sign}\,(H_i - H_J)\sqrt{\frac{2g\,|H_i - H_J|}{\lambda L/D}},$$

where Q_{iJ} is the flow from reservoir i to junction J.

(Ans: Pipeline 1: $1.20\,\mathrm{m}^3\,\mathrm{s}^{-1}$ from the reservoir to the junction, Pipeline 2: $0.33\,\mathrm{m}^3\,\mathrm{s}^{-1}$ from junction to the reservoir, and Pipeline 3: $0.873\,\mathrm{m}^3\,\mathrm{s}^{-1}$ from junction to reservoir)

Initially we assume an elevation of the total energy line at junction J: $20\,\mathrm{m}$, and to calculate the friction coefficients we assume fully rough flow in the Haaland formula:

$$\lambda \approx \frac{1}{1.8^2}\,\log_{10}^{-2}\left(\left(\frac{\varepsilon}{3.7}\right)^{10/9} + \frac{6.9\nu\pi D}{4\,|Q|}\right)$$

$$\begin{aligned}
\text{Fully rough pipe: } \lambda_1 &\approx \frac{1}{1.8^2}\,\log_{10}^{-2}\left(\left(\frac{0.0002}{3.7}\right)^{10/9}\right) = 0.0137 \\
Q_{1J} &= +0.785\sqrt{\frac{2\times 9.8\times|30-20|}{0.0137\times 3000/1}} = 1.714 \\
\text{Fully rough pipe: } \lambda_2 &\approx \frac{1}{1.8^2}\,\log_{10}^{-2}\left(\left(\frac{0.002}{3.7}\right)^{10/9}\right) = 0.0234 \\
Q_{2J} &= -0.159\sqrt{\frac{2\times 9.8\times|18-20|}{0.0234\times 600/0.45}} = -0.178 \\
\text{Fully rough pipe: } \lambda_3 &\approx \frac{1}{1.8^2}\,\log_{10}^{-2}\left(\left(\frac{0.001}{3.7}\right)^{10/9}\right) = 0.020 \\
Q_{3J} &= -0.283\sqrt{\frac{2\times 9.8\times|9-20|}{0.020\times 1000/0.6}} = -0.720 \\
Q_{1J}+Q_{2J}+Q_{3J} &= 1.714-0.178-0.720 = 0.816
\end{aligned}$$

Hence flow is accumulating at the junction – we assume a higher value of head there, $H_J = 25\,\mathrm{m}$

$$\begin{aligned}
\lambda_1 &\approx \frac{1}{1.8^2}\,\log_{10}^{-2}\left(\left(\frac{0.0002}{3.7}\right)^{10/9} + \frac{6.9\times10^{-6}\times\pi\times 1}{4\times|1.714|}\right) = 0.0141 \\
Q_{1J} &= +0.785\sqrt{\frac{2\times 9.8\times|30-25|}{0.0141\times 3000/1}} = 1.195 \\
\lambda_2 &\approx \frac{1}{1.8^2}\,\log_{10}^{-2}\left(\left(\frac{0.002}{3.7}\right)^{10/9} + \frac{6.9\times10^{-6}\times\pi\times 0.159}{4\times|-0.178|}\right) = 0.0235 \\
Q_{2J} &= -0.159\sqrt{\frac{2\times 9.8\times|18-25|}{0.0235\times 600/0.45}} = -0.333 \\
\lambda_3 &\approx \frac{1}{1.8^2}\,\log_{10}^{-2}\left(\left(\frac{0.001}{3.7}\right)^{10/9} + \frac{6.9\times10^{-6}\times\pi\times 0.283}{4\times|-0.720|}\right) = 0.0197 \\
Q_{3J} &= -0.283\sqrt{\frac{2\times 9.8\times|9-25|}{0.0197\times 1000/0.6}} = -0.875 \\
Q_{1J}+Q_{2J}+Q_{3J} &= 1.195-0.333-0.875 = -0.013
\end{aligned}$$

Now the flow balance is better. We will use linear extrapolation to estimate the head at J required to bring this to zero:

$$\begin{aligned}
\frac{H_J^{\text{refined}} - H_J^{(2)}}{0-\delta Q^{(2)}} &= \frac{H_J^{(1)} - H_J^{(2)}}{\delta Q^{(1)} - \delta Q^{(2)}} \\
H_J^{\text{refined}} &= H_J^{(2)} - \delta Q^{(2)}\,\frac{H_J^{(2)} - H_J^{(1)}}{\delta Q^{(2)} - \delta Q^{(1)}} \\
&= 25 - (-0.013)\times\frac{25-20}{-0.013-0.816} \\
&= 24.9
\end{aligned}$$

It would seem that our guess was close enough, and we will not refine it further here with another iteration, however we can refine the discharges using the same method of linear extrapolation to a zero imbalance:

$$\begin{aligned}
Q_{1J}^{\text{refined}} &= Q_{1J}^{(2)} - \delta Q^{(2)} \frac{Q_{1J}^{(2)} - Q_{1J}^{(1)}}{\delta Q^{(2)} - \delta Q^{(1)}} = 1.195 - (-0.013) \times \frac{1.195 - 1.714}{-0.013 - 0.816} = 1.203 \\
Q_{2J}^{\text{refined}} &= -0.333 - (-0.013) \times \frac{-0.333 - (-0.178)}{-0.013 - 0.816} = -0.331 \\
Q_{3J}^{\text{refined}} &= -0.875 - (-0.013) \times \frac{-0.875 - (-0.72)}{-0.013 - 0.816} = -0.873
\end{aligned}$$

Hence the flows are: Pipeline 1: $1.20\,\mathrm{m}^3\,\mathrm{s}^{-1}$ from the reservoir to the junction, Pipeline 2: $0.33\,\mathrm{m}^3\,\mathrm{s}^{-1}$ from junction to the reservoir, and Pipeline 3: $0.873\,\mathrm{m}^3\,\mathrm{s}^{-1}$ from junction to reservoir.