Hydraulics

Solution Sheet 11 – Pumps and turbines

1. *(Douglas, Gasiorek, & Swaffield, Example 23.1, p754) A centrifugal pump with impeller diameter* $0.5\,\mathrm{m}$*, when running at* 750 *revolutions/min, ("rpm", widely used in English) was tested and gave the following characteristics:*

Q $\mathrm{m^3/min}$	0	7	14	21	28	35	42	49	56
H (m)	40.0	40.6	40.4	39.3	38.0	33.6	25.6	14.5	0.0
η (%)	0	41	60	74	83	83	74	51	0

Predict the performance of a geometrically-similar pump of $0.35\,\mathrm{m}$ *diameter and running at* 1450 *rpm at each of those values of discharge. Plot these characteristics on a* $(Q, H\&\eta)$ *figure, and sketch a smooth curve for each series.*

To plot the efficiency curve it will be necessary to assume that for each Q_2 (of the second pump), the corresponding efficiency will be equal to the value at the corresponding Q_1. This is supposedly proved in DG&S, p753, but the lecturer believes that their argument is circular and he was unwilling to include it in the lecture notes. For the purpose of this example, as we have no other information on efficiency, we will have to assume that $\eta(Q_1) = \eta(Q_2)$ for every point considered.

We have $Q/ND^3 =$ constant, hence

$$\begin{aligned}
\frac{Q_2}{N_2 D_2^3} &= \frac{Q_1}{N_1 D_1^3}, \\
Q_2 &= Q_1 \times \frac{N_2}{N_1} \times \left(\frac{D_2}{D_1}\right)^3 = Q_1 \times \frac{1450}{750} \times \left(\frac{0.35}{0.5}\right)^3 \\
&= 0.663\, Q_1.
\end{aligned}$$

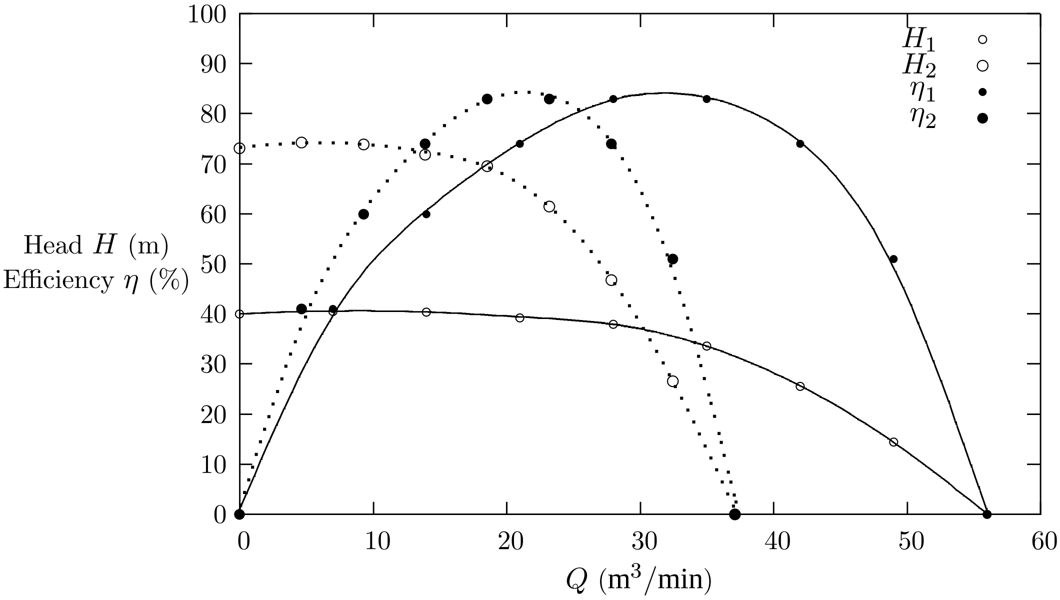
Also we have $gH/N^2D^2 =$ constant, hence

$$\begin{aligned}
\frac{H_2}{N_2^2 D_2^2} &= \frac{H_1}{N_1^2 D_1^2}, \\
H_2 &= H_1 \frac{N_2^2 D_2^2}{N_1^2 D_1^2} = H_1 \times \left(\frac{N_2}{N_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2 = H_1 \times \left(\frac{1450}{750}\right)^2 \left(\frac{0.35}{0.5}\right)^2 \\
&= 1.83\, H_1
\end{aligned}$$

Applying these to the values in the table above, and with $\eta_2 = \eta_1$ gives with the results shown in

Q $\mathrm{m^3/min}$	$0 \times 0.663 = 0$	4.64	9.28	13.9	18.6	23. 2	27. 8	32. 5	37. 1
H (m)	$40.6 \times 1.83 = 73.2$	74. 3	73. 9	71. 9	69. 5	61. 5	46. 8	26. 5	0.0
η (%)	0	41	60	74	83	83	74	51	0

the figure below.



2. *Consider a pump of characteristic diameter $D = 0.4\,\mathrm{m}$ and rotational speed 1500 revolutions/min having the following data:*

Q_1 m^3 s^{-1}	0.05	0.10	0.15	0.2	0.25
H_1 (m)	77.8	71.0	60.0	45.0	18.0
η (%)	66	79	78	60	12

a. *Calculate the power required from the motor at each discharge.*

b. *Predict the performance of a geometrically-similar pump of $0.75\,\mathrm{m}$ diameter and running at 720 rpm.*

c. *Plot these characteristics on a $(Q, H\&P\&\eta)$ figure, and sketch a smooth curve for each series.*

d. *A motor of what power would be necessary to drive that pump for a discharge of about $0.55\,\mathrm{m^3\,s^{-1}}$?*

(a) Power input to water $P = \rho QgH$, hence power from motor $P = \rho QgH/\eta$, giving successive values of 58, 88, 113, 147 and 368 kW.

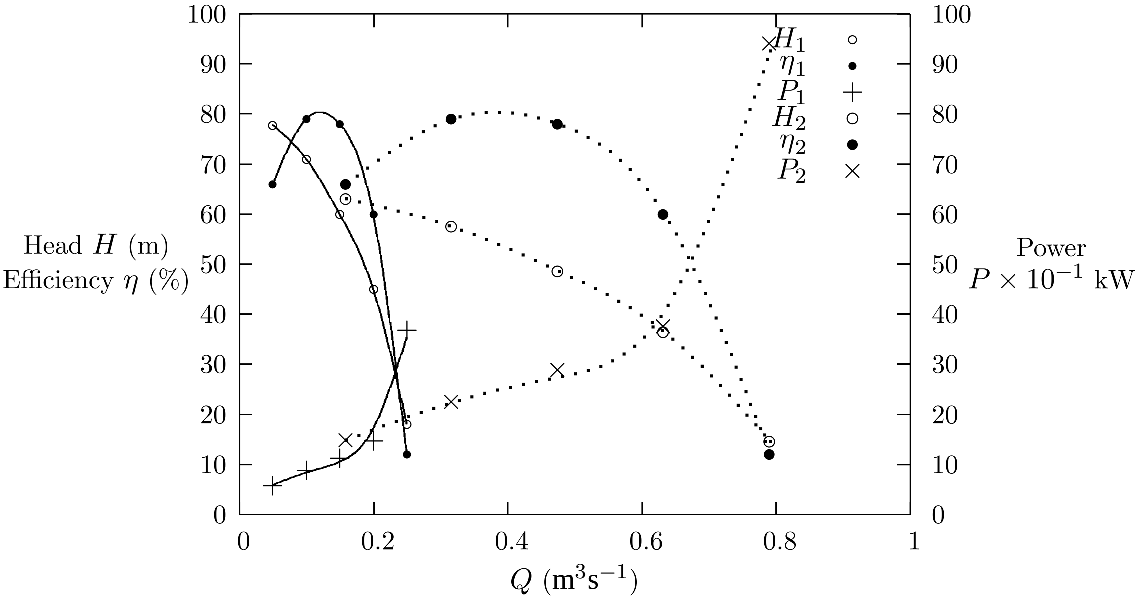
(b)

$$\begin{aligned} Q_2 &= Q_1 \times \frac{N_2}{N_1} \times \left(\frac{D_2}{D_1}\right)^3 = Q_1 \times \frac{720}{1500} \times \left(\frac{0.75}{0.4}\right)^3 = 3.16\,Q_1. \\ H_2 &= H_1 \times \left(\frac{N_2}{N_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2 = H_1 \times \left(\frac{720}{1500}\right)^2 \left(\frac{0.75}{0.4}\right)^2 = 0.81\,H_1 \\ P_2 &= 3.16\ \times 0.81 P_1 = 2.56\,P_1 \end{aligned}$$

(b) Hence we have the following table

Q_1 m^3 s^{-1}	0.05	0.10	0.15	0.2	0.25
H_1 (m)	77.8	71.0	60.0	45.0	18.0
η (%)	66	79	78	60	12
P_1 (kW)	58	88	113	147	368
Q_2 m^3 s^{-1}	0.16	0.32	0.47	0.63	0.79
H_2 (m)	63	58	49	36	15
P_2 (kW)	148	225	289	376	941

(c)



(d) From the graph of power required for the second pump, for $0.55\,\mathrm{m^3\,s^{-1}}$, about 300 kW would be required.