# Hydraulics

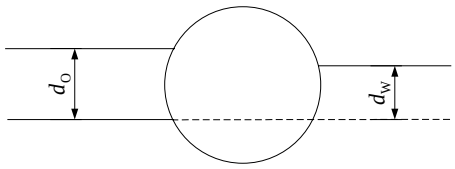
## Tutorial Sheet 3 – Forces on submerged surfaces

1. A concrete dam with a vertical upstream face impounds water of density $\rho$ to a total depth of $d$, with gravitational acceleration $g$.
   a. Sketch a graph showing the variation of pressure with elevation.
   b. Calculate the force on the dam per unit length.
   c. Calculate the depth of the centre of pressure, and add it to your sketch, showing the resultant of the force.

   (Ans.: These answers are very common in a variety of hydraulics problems. (b) $\rho g d^2/2$, you can use this result directly in problem 3 below, (c) $2d/3$ below the surface.)

2. Find the resultant force and the centre of pressure on
   a. A vertical square plate of $1.8\,\mathrm{m}$ sides, the centre of the plate $1.2\,\mathrm{m}$ below the surface. (Ans.: $38.1\,\mathrm{kN}$, $1.43\,\mathrm{m}$ below the surface)
   b. A vertical circular plate of $1.8\,\mathrm{m}$ diameter, the centre of the circle $1.2\,\mathrm{m}$ below the surface. (Ans.: $29.9\,\mathrm{kN}$, $1.37\,\mathrm{m}$.)

3. The cylindrical boom shown is required to retain a spill of oil of depth $d_\mathrm{o}$ and relative density $\sigma_\mathrm{o}$ by floating in seawater of relative density $\sigma_\mathrm{w}$.

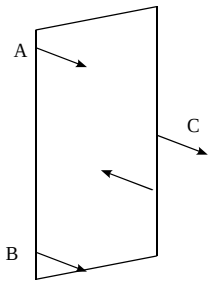



   a. Show that the seawater on the other side of the boom rises to a depth of $d_\mathrm{w} = d_\mathrm{o}\sigma_\mathrm{o}/\sigma_\mathrm{w}$ above the oil-water interface.
   b. Show that the horizontal force on the boom per unit length is, where $\rho$ is the density of fresh water:

$$\tfrac{1}{2}\rho\sigma_\mathrm{o} g d_\mathrm{o}^2 \left(1 - \sigma_\mathrm{o}/\sigma_\mathrm{w}\right),$$

   c. In which direction is the force? What implication does this have for the stability of such flexible booms in retaining oil spills? What plan shape do you think a flexible boom would take up?

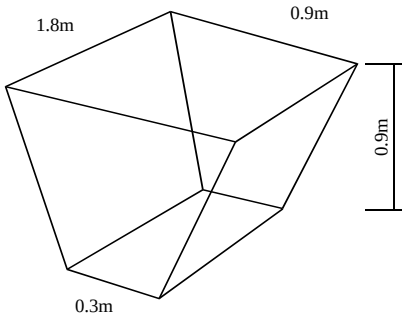
4.




   A vertical bulkhead in a ship has a door which has to be designed such that it will not be forced open if part of the ship fills with seawater of density $1025\,\mathrm{kg\,m^{-3}}$. The door is $2\,\mathrm{m}$ high and $1\,\mathrm{m}$ wide, and the sea surface is assumed to be $1\,\mathrm{m}$ above the top of the door.
   a. Calculate the force on the door and its position relative to the top of the door. (*Ans*: $40.2\,\mathrm{kN}$, $1.167\,\mathrm{m}$ below the top of the door, at the vertical centreline).

b. The door is fastened by two hinges A and B on one vertical edge, $15\,\mathrm{cm}$ from top and bottom, and by a latch C in the centre of the other vertical edge. Calculate the forces on each hinge and on the latch when one face of the bulkhead is subject to water pressure (*Ans.*: $20.1\,\mathrm{kN}$, $6.1\,\mathrm{kN}$, $14.0\,\mathrm{kN}$).

5.




A mixing tank with trapezoidal ends is $1.8\,\mathrm{m}$ long, $0.9\,\mathrm{m}$ deep and $0.9\,\mathrm{m}$ wide at the top, tapering to $0.3\,\mathrm{m}$ wide at the base. If the tank is completely filled with water, calculate

a. The total weight force of water in the tank.

b. The total force exerted by water on the base.

c. The total force exerted on one end and where it acts. To do this it will be necessary to calculate the first and second moments of area of the trapezium which forms the face about an axis along the top of the trapezium, the water level.

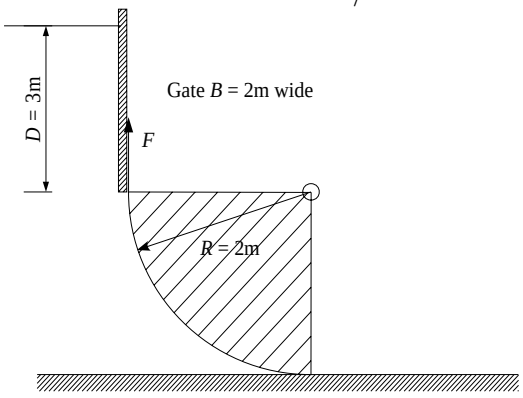
   i. The first moment of area $M_\mathrm{O} = A\bar{h}$ can be calculated by using known properties of rectangles and triangles. It is suggested that you calculate it that way ($A\bar{h} = 0.2025\,\mathrm{m}^3$).

   ii. Then, to check – and to prepare for the next part – set up and evaluate an integral for that first moment of area in terms of the width $b$ of the trapezium as a function of depth $h$.

   iii. Then, set up and evaluate a similar integral for the second moment of area $I_\mathrm{O}$, so that you can use the expression $h_\mathrm{CP} = I_\mathrm{O}/M_\mathrm{O}$.

(*Ans.*: $9.5\,\mathrm{kN}$, $4.8\,\mathrm{kN}$, $2.0\,\mathrm{kN}$, $0.54\,\mathrm{m}$ below top).

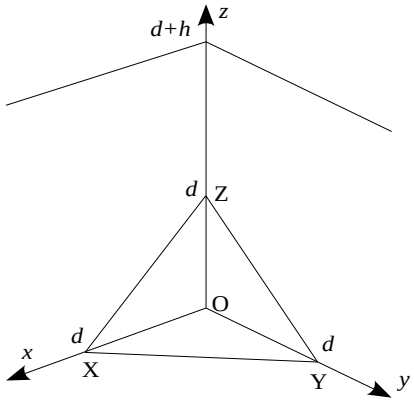
6. A sluice gate consists of a radial gate of radius $2\,\mathrm{m}$ pivoted at its centre O, as shown in the figure. Calculate the magnitude and direction of the resultant force on the gate due to the water, and the net moment required to open the gate by calculating the moments of the horizontal and vertical forces. (*Ans.*: Horizontal force $157\,\mathrm{kN}$, $1.083\,\mathrm{m}$ below the pivot, Vertical force $179\,\mathrm{kN}$ at a horizontal distance of $0.948\,\mathrm{m}$ from the pivot, resultant $238\,\mathrm{kN}$, at an angle to the horizontal of $49°$, nett moment $\approx 0$ (not exactly because of roundoff errors in these figures.) You may need the result that the centroid of a semicircular disk of radius $a$ is at a distance of $4a/3\pi$ from the straight side of the disk..




7. A spherical container is made up of two hemispheres, the joint between the two halves being horizontal. The sphere is completely filled with water through a small hole in the top. It is found that

50kg of water are required for this purpose. If the two halves of the container are not secured together, what must be the mass of the upper hemisphere if it just fails to lift off the lower hemisphere? (*Ans.*: 12.5 kg).

8. The corner of a tank is bevelled by equal dimensions $d$ as shown in the figure.




The tank is filled to $h$ above the top of the bevel. Show that each component of the force on the triangular corner is

$$\frac{\rho g d^2}{6}\left(3h + 2d\right).$$