Hydraulics

Tutorial Sheet 8 – Dimensional analysis and similitude

1. Verify that the following expressions are dimensionally homogeneous:
   a. the integral energy expression for a length of pipe or channel
   $$-Q_{\text{in}}\left(\frac{p}{\rho g}+z+\frac{\alpha}{2g}\frac{Q^2}{A^2}\right)_{\text{in}}+Q_{\text{out}}\left(\frac{p}{\rho g}+z+\frac{\alpha}{2g}\frac{Q^2}{A^2}\right)_{\text{out}}=0,$$
   b. and the momentum expression for a force of fluid on a control surface
   $$\mathbf{P}=\sum_j\left(\rho\beta_j\frac{Q_j^2}{A_j}+\bar{p}_jA_j\right)(-\hat{\mathbf{n}}_j)+\mathbf{F}_{\text{body}}.$$

2. Assuming that the thrust F of a ship's propeller is dependent upon the diameter d, speed of advance V, fluid density ρ, revolutions per second n and coefficient of viscosity μ,
   a. Show that the thrust can be expressed by the equation
   $$\frac{F}{\rho V^2d^2}=f\left(\frac{\mu}{\rho Vd},\frac{dn}{V}\right),$$
   b. Describe the physical significance of each of the terms in the equation.

3. The variables controlling the motion of a ship through water are the drag force F, the speed U, the length l, the fluid density ρ, dynamic viscosity of the water μ, and gravitational acceleration g.
   a. Show that the force must be given by an expression of the form
   $$\frac{F}{\rho U^2l^2}=f\left(\frac{\mu}{\rho Ul},\frac{gl}{U^2}\right),$$
   where $f()$ is the functional dependence which is to be determined experimentally.
   b. Interpret each of the terms in the equation.
   c. A model boat of a scale $1:50$ was measured in a towing tank to have a drag force of $0.27\,\text{N}$ when towed at a velocity of $0.91\,\text{m s}^{-1}$.
      i. If it is assumed that most drag is not due to friction but to the waves generated, estimate the drag force on the protoype boat. (*Ans*: $33.8\,\text{kN}$)
      ii. Estimate the power required to propel the boat. (*Ans*: $217\,\text{kW}$)
   d. How might the expression in (a) be modified if the ship is in water of finite depth D?

4. A V-notch weir is a vertical plate with a notch of angle θ cut into the top of it and placed across an open channel. The discharge Q through the notch is a function of the elevation H of the upstream liquid surface above the bottom of the notch. The discharge also depends upon gravity g and upon the velocity of approach V_0 to the weir.
   a. Use the Buckingham Π theorem to show that there are three governing dimensionless parameters, and that the discharge equation may be expressed in the form:
   $$\frac{Q}{(gH^5)^{1/2}}=f(V_0/(gH)^{1/2},\theta),$$
   where $f()$ is the functional dependence which is to be determined experimentally.
   b. In practice it is found that the approach velocity is not very important. Ignore the effect of that velocity to estimate the ratio of discharge in the full scale weir to that in a geometrically-similar laboratory weir at a scale of $1:10$. (*Ans*: 316)