Worked solution - Tutorial Sheet 1

```
restart;read "C:/JF/Software/Maple/Start.mpl";Digits:=4:
```

Question 1 - Strickler

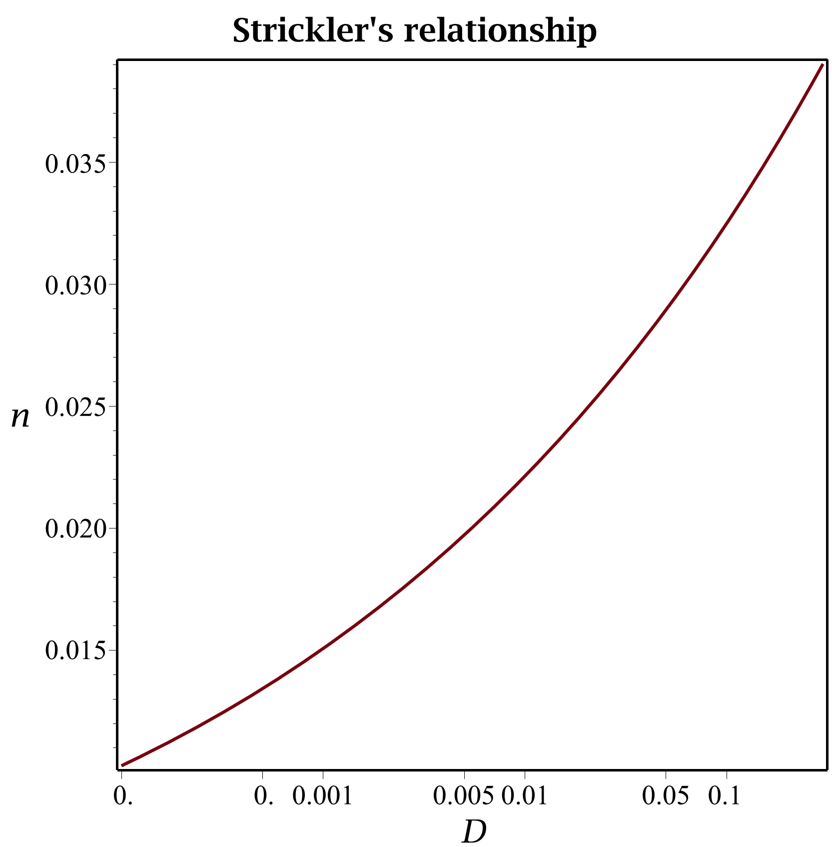
```
n:=D^(1./6.)/6.7/sqrt(g);
```

$$n := \frac{0.1493\,D^{0.1667}}{\sqrt{g}} \tag{1.1}$$

```
g:=9.8;
```

$$g := 9.8 \tag{1.2}$$

```
with(plots,semilogplot):semilogplot(n,D=0.1/1000..0.3,title="Strickler's relationship",
legend="",labels=[D,"n"],labelfont=[Times,italic,12],titlefont=[Times,bold,12],axes=
boxed);
```



Now consider the difference between a 1cm grain and a 2cm grain:

```
subs(D=0.01,n);subs(D=0.02,n);
```

$$0.02214$$

$$0.02485 \quad (1.3)$$

For an increase of size of 100%, an increase of n of only 10%. We do not have to be too worried about how accurately we know the size.

```
n:='n':
```

Question 2

```
B:=W+2*m*h;A:=h*(W+m*h);P:=W+2*sqrt(1.+m*m)*h;
```

$$B := W + 2\, m\, h$$

$$A := h\,(W + m\, h)$$

$$P := W + 2\sqrt{1. + m^2}\; h \quad (2.1)$$

Check:

```
diff(A,h);
B;
```

$$W + 2\, m\, h$$

$$W + 2\, m\, h \quad (2.2)$$

Now for the first moment, we use the fact that the centroid of a triangle is 1/3 of the distance from the base

```
M:=W*h*h/2+2*1/2*m*h*h*h/3;
```

$$M := \frac{1}{2}\, W h^2 + \frac{1}{3}\, m\, h^3 \quad (2.3)$$

```
diff(M,h);
expand(A);
```

$$W h + m\, h^2$$

$$W h + m\, h^2 \quad (2.4)$$

Question 3

```
Q=1/n*A^(5/3)/P^(2/3)*sqrt(S);
```

$$Q = \frac{(h\,(W + m\, h))^{5/3}\sqrt{S}}{n\left(W + 2\sqrt{1. + m^2}\; h\right)^{2/3}} \quad (3.1)$$

Question 4

(e) Chezy-Weisbach

```
A:='A':P:='P':g:='g':Q=sqrt(`8`*g/lambda)*A^(3/2)/P^(1/2)*sqrt(S);
```

$$Q = \frac{\sqrt{\frac{8\,g}{\lambda}}\,A^{3/2}\,\sqrt{S}}{\sqrt{P}} \tag{4.1.1}$$

Divide both sides by $h^{\frac{3}{2}}$ $\frac{Q}{h^{\frac{3}{2}}} = \frac{\sqrt{\frac{`8`\,g}{\lambda}}\left(\frac{A}{h}\right)^{\frac{3}{2}}\sqrt{S}}{P^{\frac{1}{2}}}$

$$\frac{Q}{h^{3/2}} = \frac{\sqrt{\frac{8\,g}{\lambda}}\left(\frac{A}{h}\right)^{3/2}\sqrt{S}}{\sqrt{P}} \tag{4.1.2}$$

Now, the $\frac{A}{h}$ on the right (approximately the width) is a relatively slowly-varying function of h. Solve for $h^{\frac{3}{2}}$ on the left

```
h = (Q/(sqrt(`8`*g/lambda)*(A/h)^(3/2)*sqrt(S)/P^(1/2)))^(2/3);
```

$$h = \left(\frac{Q\sqrt{P}}{\sqrt{\frac{8\,g}{\lambda}}\left(\frac{A}{h}\right)^{3/2}\sqrt{S}}\right)^{2/3} \tag{4.1.3}$$

```
h=Q^(2/3)/(`8`*g/lambda)^(1/2*2/3)/(A/h)/(S)^(1/3)*P^(1/3);
```

$$h = \frac{Q^{2/3}\,h\,P^{1/3}}{\left(\frac{8\,g}{\lambda}\right)^{1/3}A\,S^{1/3}} \tag{4.1.4}$$

```
h[1]:=(lambda*Q^2/`8`/g/S)^(1/3)*P^(1/3)/(`A/h`);
```

$$h_1 := \frac{\left(\frac{\lambda\,Q^2}{8\,g\,S}\right)^{1/3}P^{1/3}}{A/h} \tag{4.1.5}$$

(f) Dimensionally-correct?

$$subs\left(Q=\frac{L^3}{T}, \lambda=1, `8`=1, g=\frac{L}{T^2}, S=1, P=L, `A/h`=\frac{L^2}{L}, h[1]\right)$$

$$\frac{\left(L^5\right)^{1/3}}{L^{2/3}} \quad \textbf{(4.2.1)}$$

... which gives a dimension of L, that of *h* on the left of the equation

Question 5

$W := 10 : m := 2 : S := 0.0005 : \mathrm{n} := 0.02 : Q := 20 :$

$B := W + 2\,m\,h; A := h\,(W + m\,h); P := W + 2\sqrt{1. + m\,m}\;h$

$$B := 10 + 4\,h$$

$$A := h\,(10 + 2\,h)$$

$$P := 10 + 4.472\,h \quad \textbf{(5.1)}$$

Initially assume top breadth = *W*: B0:=W;

$$B0 := 10 \quad \textbf{(5.2)}$$

Use the formula in the notes - to get first estimate of *h* from wide channel approximation:

$H[0] := \left(\frac{n\,Q}{B0\sqrt{S}}\right)^{3.\frac{1}{5}}$

$$H_0 := 1.418 \quad \textbf{(5.3)}$$

Apply the iteration formula:

for i **from** 0 **to** 3 **do**

$$h := H[i];\ H[i+1] := \frac{\left(\frac{n\,Q}{\sqrt{S}}\right)^{\frac{3}{5}} P^{\frac{2}{5}}\,h}{A};\ print(evalf(H[i+1], 4))$$

end do :

$$1.343$$

$$1.349$$

$$1.348$$

$$1.348 \quad \textbf{(5.4)}$$