Consider a trapezoidal channel of bed width $10\,\mathrm{m}$ *and batter slopes* $2:1$ *(H:V) excavated to a slope of 1 in 10,000, and Manning's* $n = 0.03$*. For a depth of* $2.5\,\mathrm{m}$*,*

1. a. *Calculate the discharge* Q *and the mean velocity in the channel*

$$\begin{aligned} B &= W + 2mh = 10 + 2\times 2\times 2.5 = 20\,\mathrm{m} \\ A &= h\,(W + mh) = 2.5\times(10 + 2\times 2.5) = 37.5\,\mathrm{m}^2 \\ P &= W + 2h\sqrt{1+m^2} = 10 + 2\times 2.5\sqrt{1+2^2} = 21.2\,\mathrm{m} \\ Q &= \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}\sqrt{S} = \frac{1}{0.03}\times\frac{37.5^{5/3}}{21.2^{2/3}}\sqrt{0.0001} = 18.3\,\mathrm{m}^3\,\mathrm{s}^{-1}, \\ U &= Q/A = 18.3/37.5 = 0.49\,\mathrm{m\,s}^{-1}. \end{aligned}$$

b. *Calculate the kinematic wave speed and also calculate it using the wide-channel approximation. Why is the latter not so accurate here? Calculate the "dynamic wave speed",* $C = \sqrt{gA/B}$*.*

i. Very long wave speed, full expression (using $dP/dh = 2\sqrt{1+m^2}$)

$$\begin{aligned} c &= \frac{5}{3}U\left(1 - \frac{2}{5}\frac{A}{PB}\frac{dP}{dh}\right) \\ &= \frac{5}{3}\times 0.49\times\left(1 - \frac{2}{5}\frac{37.5}{21.2\times 20}\times 2\sqrt{5}\right) \\ &= 0.69\,\mathrm{m\,s}^{-1} \end{aligned}$$

ii. Kinematic wave speed, wide channel approximation

$$c = \frac{5}{3}\frac{Q}{A} = \frac{5}{3}\times 0.49 = 0.82\,\mathrm{m\,s}^{-1}$$

The approximation is not very accurate – for this example where $h/W \approx 0.25$ is not small.

iii. Dynamic wave speed:

Conventional expression

$$C = \sqrt{\frac{gA}{B}} = \sqrt{\frac{9.8\times 37.5}{20}} = 4.3\,\mathrm{m\,s}^{-1}$$

Also, let's just use the more complete expression given in the lecture notes, and use $\beta = 1.2$:

$$C = \sqrt{\frac{gA}{B} + \frac{Q^2}{A^2}\left(\beta^2 - \beta\right)} = \sqrt{\frac{9.8\times 37.5}{20} + 0.49^2\times(1.2^2 - 1.2)} = 4.3\,\mathrm{m\,s}^{-1},$$

and we see that the correction for the velocity distribution had very little effect. This is because its importance is measured by the square of the Froude number. In our case, $\mathsf{F}^2 = Q^2B/gA^3 = 18.3^2\times 20/9.8/37.5^3 = 0.013$, which is small, and the correction term is $0.5\times 0.013\times(1.44 - 1.2) = 0.0016$. However, for flows with large Froude number, and especially supercritical flows, this term will be important.

c. *Calculate the estimated time of travel of a disturbance over a distance of* $10\,\mathrm{km}$*. (Ans:* $4\,\mathrm{h}$*).*

$$T = \frac{L}{c} = \frac{10000}{0.69} \approx 14500\,\mathrm{s} = \frac{14500}{3600} = 4.0\,\mathrm{h}$$