Institute of Hydraulic and Water Resources Engineering, Vienna University of Technology

River Engineering

Tutorial Sheet 3 – The long wave equations

1. Write a short essay, possibly in bullet point form and possibly without mathematics, describing the approximations and operations required to obtain the long wave equations

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = i, \tag{1a}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\beta \frac{Q^2}{A}\right) + \frac{gA}{B}\frac{\partial A}{\partial x} = gA\tilde{S} - \Omega Q\,|Q|\,. \tag{1b}$$

2. Describe the physical significance of each of the terms in the equations.

3. The speed at which the main body of disturbances move in rivers is the *very long wave speed* c. The Kleitz-Seddon formula for c is

$$c = \frac{dQ_\mathrm{r}}{dA} = \frac{1}{B}\frac{dQ_\mathrm{r}}{d\eta},$$

where Q_r is a function of area A or water surface elevation η respectively, given by any of the expressions

$$Q_\mathrm{r} = \begin{cases} Q_\mathrm{r} & \text{measured;} \\ \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}\sqrt{S} = k_\mathrm{St}\frac{A^{5/3}}{P^{2/3}}\sqrt{S}, & \text{Gauckler-Manning ;} \\ A^{3/2}\sqrt{\frac{g}{\Lambda P}S}, & \text{Chézy-Weisbach .} \end{cases}$$

   a. Show that the value of c, using the Gauckler-Manning expression, can be written in terms of the mean velocity of flow, $U = Q/A$ as

$$c = U\left(\frac{5}{3} - \frac{2}{3}\frac{A}{P}\frac{dP}{dA}\right).$$

   b. It is more convenient to express the derivative in terms of the local water height. Using the fact that $dA/dh = B$ (interpret that physically) show that

$$c = \frac{5}{3} \times U \times \left(1 - \frac{2}{5}\frac{A}{PB}\frac{dP}{dh}\right).$$

   c. For a trapezoidal section show that the quantity $dP/dh = 2\sqrt{1+m^2}$, where m is the batter slope (H:V), so that this might have a value of, for $m = 2$ say, of $2\sqrt{5} \approx 4.5$. This means that the relative contribution due to the perimeter changing is roughly $2 \times A/BP$, which is the twice the mean depth divided by the wetted perimeter, which will be small for typical wide shallow channels.

4. Consider a trapezoidal canal of bed width $10\,\mathrm{m}$ and batter slopes $2:1$ (H:V) excavated to a slope of 1 in $10\,000$, and $n = 1/k_\mathrm{St} = 0.03$. For a depth of $2.5\,\mathrm{m}$,

   a. Calculate the discharge Q and the mean velocity in the channel (*Ans:* $Q = 18.3\,\mathrm{m^3\,s^{-1}}$, $U = 0.49\,\mathrm{m\,s^{-1}}$).

   b. Calculate the very long wave speed and also calculate it using the wide-channel approximation. Calculate the "dynamic wave speed", $C = \sqrt{gA/B}$. (*Ans:* $c = 0.69\,\mathrm{m\,s^{-1}}$, $0.81\,\mathrm{m\,s^{-1}}$, $C = 4.3\,\mathrm{m\,s^{-1}}$).

   c. Calculate the estimated time of travel of a disturbance over a distance of $10\,\mathrm{km}$. (*Ans:* $4\,\mathrm{h}$).