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Modelling phosphorus transport in a surface irrigation drain

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Abstract

Understanding phosphorus (P) transport from agricultural land is essential to the development of effective management strategies that reduce the impact of agriculture on inland water quality. This paper describes the development and application of a process based model describing P transfer down a farm scale irrigation drain.

The model uses a volume routing equation combined with the Kostiakov infiltration equation to route water down the drain. Using estimated infiltration parameters and literature values of Manning's Roughness the flow-model predicted the total volume of water transfer down a 180 m long irrigation drain within 5%, with flow rates predicted within 10%.

The flow was then used to provide input data for modelling P transfer down the drain. The P model combines a simple advective equation with rate equations to describe P release by plants and the uptake and release of P by bed sediments. Data from four field investigations were used to parameterise the model with concentrations and loads predicted within 5%.

Applied over 14 months, the results from four modelled management scenarios suggest that a bare earth drain, or an intermittently cleared drain may reduce P export over 180 m by 9–19%. While the cost of current management strategies is likely to ultimately limit implementation on commercial farms, the model presented in this paper provides a useful basis for the investigation of further management scenarios, which may reduce P export from irrigated dairy farms.

Keywords: Surface drains; Ditches; Volume routing; Elovich equation

1. Introduction

Inland water systems have an inherent value, environmentally and to the community through fishing, tourism, recreation and their aesthetic appeal. Phosphorus (P) contributes to eutrophication and the development of algal blooms, which negatively affect these important assets [2,20,50]. Phosphorus entering these inland water systems comes from a range of sources within a catchment, including dryland and irrigated agriculture. For example, of the P entering the Gippsland lakes south-eastern Australia, an estimated 53 t (23%) is from the Macalister Irrigation District [19]; 53,000 ha of farm land which is dominated by irrigated perennial pastures for dairy production.

Research into P export from agricultural land has focussed predominantly on the paddock (field) scale [18,24,38], defined as the smallest farm management unit. While P mobilisation and transport at the paddock scale is important, P exported from paddocks only has

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an effect on inland water systems if it is transported through the catchment. Concentrations and loads of P are often an order of magnitude less in streams than paddock scale investigations would suggest. For example, in the Macalister Irrigation District southeastern Australia (58,000 ha) a decrease in P loads in the drainage networks of between 5:1 and 15:1 can be estimated from paddock and catchment scale loads [6]. Similarly in the Goulburn Broken catchment (northern Victoria, Australia) ratios of between 2:1 and 22:1 have been estimated for P loads between the paddock and catchment scales [28]. Understanding the transport of P between the paddock and catchment scales is therefore important for the development of effective management strategies.

Farms represent an important link between the paddock and catchment scale, as they are the largest single management unit in a catchment and a likely scale at which regulatory conditions on P export may be set. On an irrigated dairy farm, surface drains (channels or ditches) are the main pathway for water and P movement [5], thus understanding P transfer through drains is important for effectively managing P export between the paddock and catchment scales.

Research has shown that surface drains can significantly alter the P concentrations in water [7,9,42,47] over a short time scale (days-weeks). However, the effect of drains on P export over an irrigation season or year cannot be determined without long term monitoring or the application of a process based mathematical model. This paper develops such a model, which describes both flow and phosphorus dynamics in an irrigation drain. The model was developed as a predictive tool to investigate the effect of different management strategies on P export down an irrigation drain. Fourteen months of field monitoring data from a farm scale irrigation drain in the Macalister Irrigation District was used to evaluate the performance of the model for water and P transport and the impact an irrigation drain might have on P transfer between the paddock and the farm boundary.

2. Modelling flow in the irrigation drain

The first step in modelling P export down an irrigation drain was to accurately describe water movement in the drain. Water movement in farm scale irrigation drains is generally intermittent, with discrete flow events occurring in response to irrigation or rainfall runoff. The selected routing equation had to adequately describe flow along a drainage reach, including the movement of the wetting front, recession at the end of flow, and infiltration throughout the flow event.

The volume routing equation, was used as the basis for the model [15]:

$$\frac{\partial V}{\partial t} + K\frac{\partial V}{\partial x}\sqrt{S - \frac{1}{B}\frac{\partial^2 V}{\partial x^2}} = \int_{x_0}^{x} q(x')\,\mathrm{d}x', \tag{1}$$

where the volume of water V upstream of point x at time t, was a function of the mean bed slope of the drain (S), the inflow (q), the surface width of the channel (B) and conveyance (K). For the purposes of this model, the volume routing Eq. (1) was modified to calculate flow per unit width of the drain:

$$\frac{\partial v}{\partial t} + k\frac{\partial v}{\partial x}\sqrt{S - \frac{\partial^2 v}{\partial x^2}} = \int_{x_0}^{x} i(x')\,\mathrm{d}x', \tag{2}$$

such that v represents the upstream volume of water per unit width ($V = vB$), which was a function of inflow and conveyance per unit width (i and k, respectively). Conveyance was a function of Manning's roughness (n) and the depth of the water column (d):

$$k = \frac{d^{5/3}}{n}. \tag{3}$$

While there were no lateral inflows into the modelled drain, infiltration was potentially a significant flow path, with the inflow component (i) a function of infiltration. Infiltration was represented by the Kostiakov equation [30], where the cumulative infiltration (Z) was a function of time and two empirical parameters (a_{inf} and r_{inf}):

$$Z = a_{\mathrm{inf}} t^{r_{\mathrm{inf}}}. \tag{4}$$

The simple form of the Kostiakov equation was in keeping with the purpose of the model and adequately represented the basic dynamics of infiltration [3]. Combining Eqs. (2) and (4) yielded the modified volume routing equation used to describe flow in a farm scale irrigation drain, providing the basis for modelling P transfer.

$$\frac{\partial v}{\partial t} + k\frac{\partial v}{\partial x}\sqrt{S - \frac{\partial^2 v}{\partial x^2}} = -\int_{x_0}^{x} \frac{\partial Z}{\partial t}\,\mathrm{d}x'. \tag{5}$$

The flow model represented by Eq. (5) had an advective or travelling wave nature [15], which supported the use of a relatively simple advective finite difference scheme. The numerical solution (described in Appendix A) was found to be

$$v_{(x,t+\Delta)} \approx v_{(x,t)} + \frac{(\Delta c_k)^2}{2}\frac{\partial^2 v}{\partial x^2} - \Delta\frac{d^{5/3}}{n}\frac{\partial v}{\partial x}\sqrt{S - \frac{\partial^2 v}{\partial x^2}} - a_{\mathrm{inf}}(t^{r_{\mathrm{inf}}} - (t+\Delta)^{r_{\mathrm{inf}}})\delta, \tag{6}$$

where Δ was the time step used in the model (~30 s), δ was the distance interval used by the model (10 m), c_k was a constant that may be described as the average kinematic wave speed [15] and $\partial v/\partial x$ and $\partial^2 v/\partial x^2$ are the first and second derivatives of v with respect to x approximated by the three-point formulae:

$$\frac{\partial v}{\partial x} = \frac{v_{(x+\delta,t)} - v_{(x-\delta,t)}}{2\delta}, \tag{7}$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{v_{(x+\delta,t)} + v_{(x-\delta,t)} - 2v_{(x,t)}}{\delta^2}. \tag{8}$$

The advective scheme used was beneficial not only in terms of solution stability, but also in terms of coupling the hydraulic equation with phosphorus transfer which was a function of advective transport and the interactions between P, bed sediments and plants.

2.1. Application of the flow model

Water and P transport down a farm scale irrigation drain was investigated on the Macalister Research Farm (Fig. 1), southeastern Australia (38°0′S, 146°54′E). The Macalister Research Farm is a community owned commercially operated dairy farm that consists of 80 ha of irrigated perennial pasture. Between December 2000 and January 2002 water movement in the drain was monitored, with management actions affecting drain condition and potentially flow. Over the experimental period the drain changed from a recently excavated (cleaned out and reformed) bare earth drain (Dec-00) through a period of plant growth (May-01), plant decay after herbicide application (Jun-01, Jul-01) and a second period of plant growth (Jan-02).

Water flowing into the top of a drain (Fig. 2) from 11.8 ha of pasture was monitored using a 300 mm RBC flume [11] and an ISCO storm monitoring system comprising a model 4230 bubbler flow meter and model 3700 automatic sampler (ISCO Inc., USA). During this 14-month period four field investigations were conducted (field investigation 1:18-Dec-00, 2:9-May-01, 3:7-Jun-01 and 4:31-Jul-01) [7], with flow monitored at the top of the drain as previously described and at the bottom of the drain using a 200 mm RBC flume and a pair of 392 capacitance probes (Dataflow Systems Pty Ltd, New Zealand).

The performance of the modified volume routing equation was determined by routing flow along the monitored drain, which was 180 m long, 3.5 m wide and had an average bed slope of 1:800. The bed sediment in the drain was a heavy clay in a soil type with low permeability [41]. Vegetation in the drain varied over the 14 months from bare earth, through a period of weed growth, followed by decay after herbicide application.

2.1.1. Input parameters

The application of the modified volume routing equation required estimates of the input parameters,

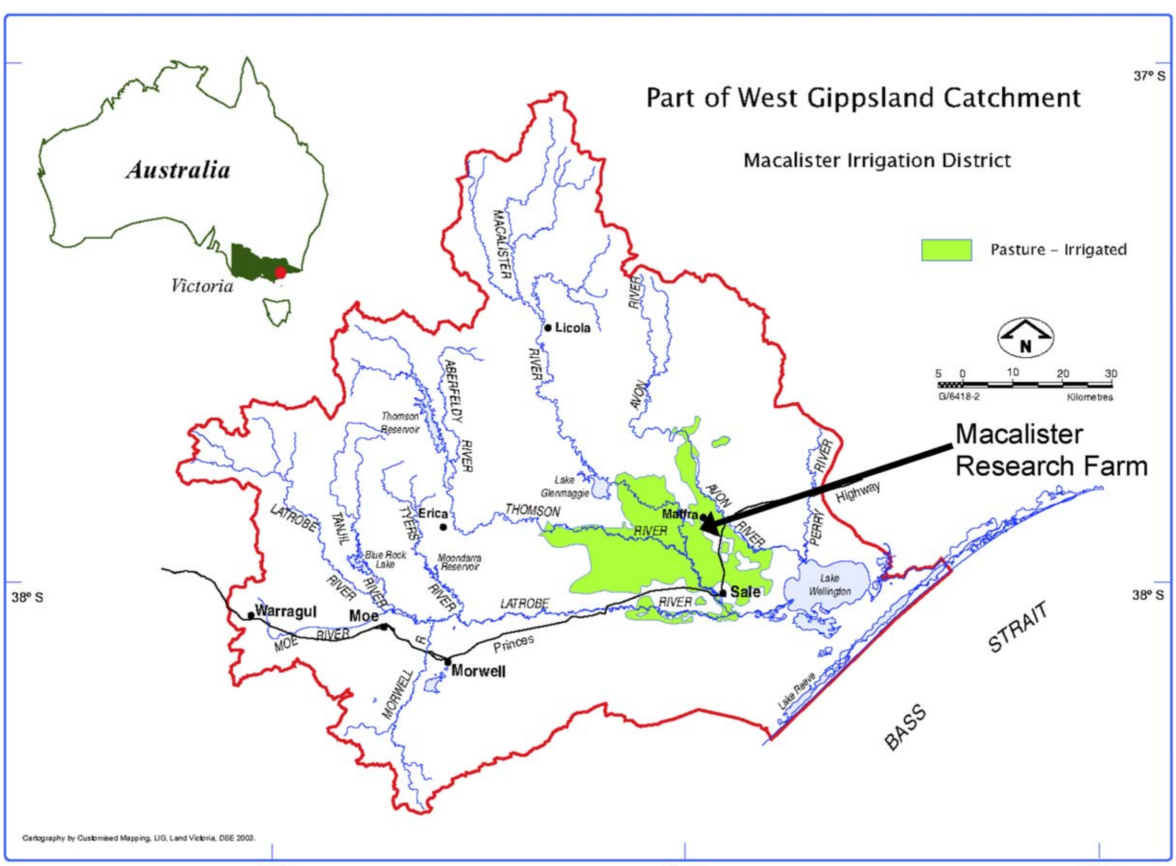

Fig. 1. Map showing the location of the Macalister Research Farm where the experimental work was conducted.

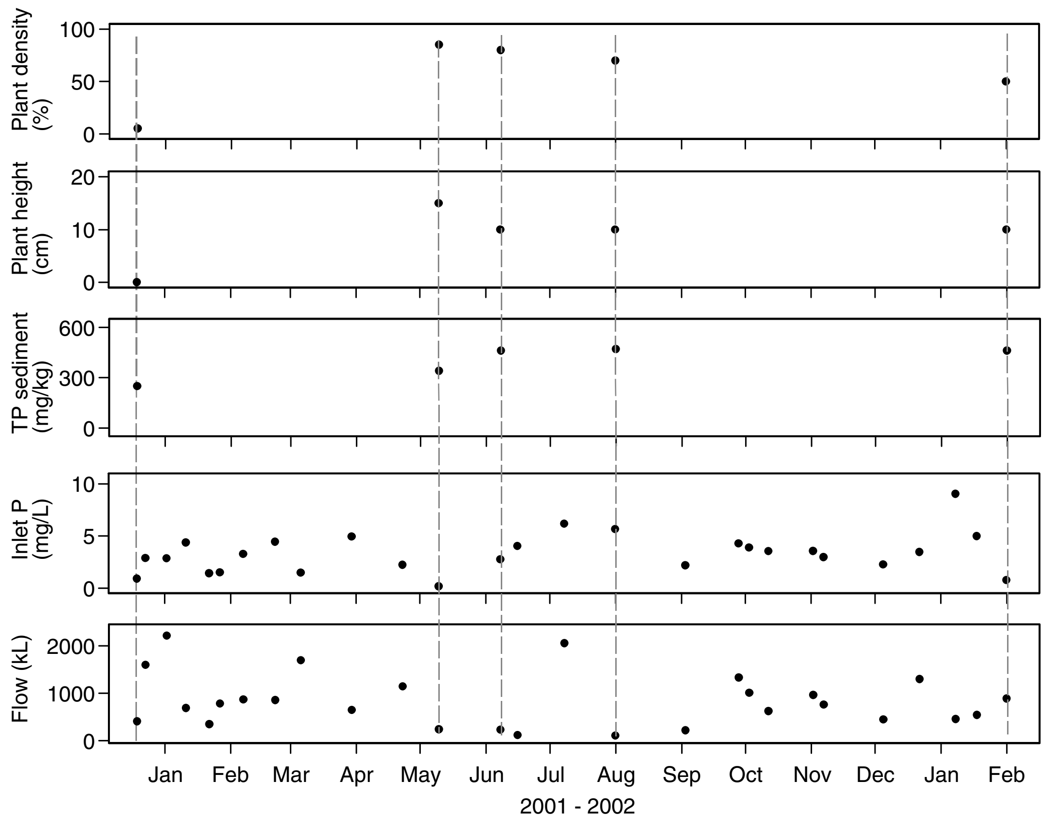

Fig. 2. Summary of sediment and plant characteristics as well as flow volumes and average inlet concentrations for field investigations and runoff events over the 14-month modelling period (vertical lines represent the timing of field investigations).

particularly Manning's roughness coefficient and infiltration parameters. With drain conditions varying over the 14-month period, parameter estimates based on measurable characteristics were required so that model parameters could be adjusted to reflect physical conditions.

Manning's roughness coefficients collected from the literature [13,17,22,31,32] were used as the basis for modeling flow in the irrigation drain. Initial drain conditions, at the start of the 14 month period, consisted of minimal plant cover (<5%) and a relatively smooth clay bed, suggesting a Manning's roughness coefficient of between 0.010 and 0.030 [13,17,32]. For model application an estimate of 0.015 was made consistent with published estimates [17], similar estimates were made for the remaining experimental flows (Table 1). A linear interpolation over time was used to estimate Manning's roughness for all irrigation and rainfall runoff events between experimental flows. Linear interpolation was used to estimate drain conditions between experimental flows, as plant cover was not measured for each runoff event. Between the experimental flows there was a gradual change in drain conditions with experimental flows marking a management action which affected drain conditions. For example, from 18-Dec-00 there was a gradual increase in plant cover (from bare earth) over time until maximum plant cover was achieved on 9-May-01 (Fig. 2).

Infiltration into the heavy clay bed sediment of the drain was expected to be low, less than 10 mm per day based on measurements of infiltration in the neighbouring irrigation bay (Barlow 2000, unpublished data). This led to initial estimates of a_{inf} and r_{inf} of 0.002 and 0.15, respectively.

Table 1
Manning's roughness coefficients (n) estimated for the farm drain for the four field investigations between Dec-00 and Jan-02

Field investigation	Date	Ground cover (%)	Plant height (cm)	Qualitative description of plant cover[a]	n[a]
1	18-Dec-2000	<5	0	<Grass sparse	0.015
2	9-May-2001	85	15	Grass—good	0.045
3	7-Jun-2001	80	10	Between grass—good and grass—fair	0.040
4	31-Jul-2001	70	10	Grass—fair	0.035

[a] Foster et al. [17].

2.1.2. Boundary conditions

Two boundary conditions were used in the prediction of water movement in the irrigation drain that defined the movement of the wetting front and recession at the end of flow:

- flow commenced when the cumulative volume of water routed into the drain was greater than the cumulative infiltration, with a minimum depth of 2 mm of water required in the drain before flow started, and
- recession or the end of flow was presumed to have occurred when the change in cumulative volume was less than 0.1 m^3 (0.1 mm change in depth) for a time step.

2.1.3. Statistical analysis

The performance of the volume routing equation (5) was evaluated using measured and predicted flow at the drain outlet. The criteria used were, (a) the relative error between measured and predicted flow volume, (b) the mean squared error of discharge, and (c) a graphical analysis of the residual values.

The total volume of water exported from the drain was analysed by calculating the relative error (ε) between the measured and predicted volumes at the end of the irrigation drain (v_{meas} and v_{pred}, respectively). A satisfactory prediction was assumed when the relative error (ε) was less than 0.05% or 5%.

$$\varepsilon = \frac{(v_{\text{meas}} - v_{\text{pred}})}{v_{\text{meas}}}. \tag{9}$$

The mean squared error (s^2) was used as a measure of the variance between measured and predicted flow rates [53], with a graph of the residual values used to highlight any skew (or bias) within the model prediction.

$$s^2 = \frac{1}{n}\sum(\text{meas} - \text{pred})^2. \tag{10}$$

The Nash and Suttcliffe Coefficient of Efficiency (COE) was also used to assess the performance of the flow model. It has been widely used as a measure of model performance in hydrology, describing the deviation from unity (one) of the ratio of the mean squared errors and the variance of the observations [40].

2.2. Flow model—results and discussion

The flow model adequately predicted the movement of water down the drain during field investigation 1 (18-Dec-00), when the estimated Manning's roughness ($n = 0.015$) and infiltration ($a_{\text{inf}} = 0.002$, $r_{\text{inf}} = 0.15$)

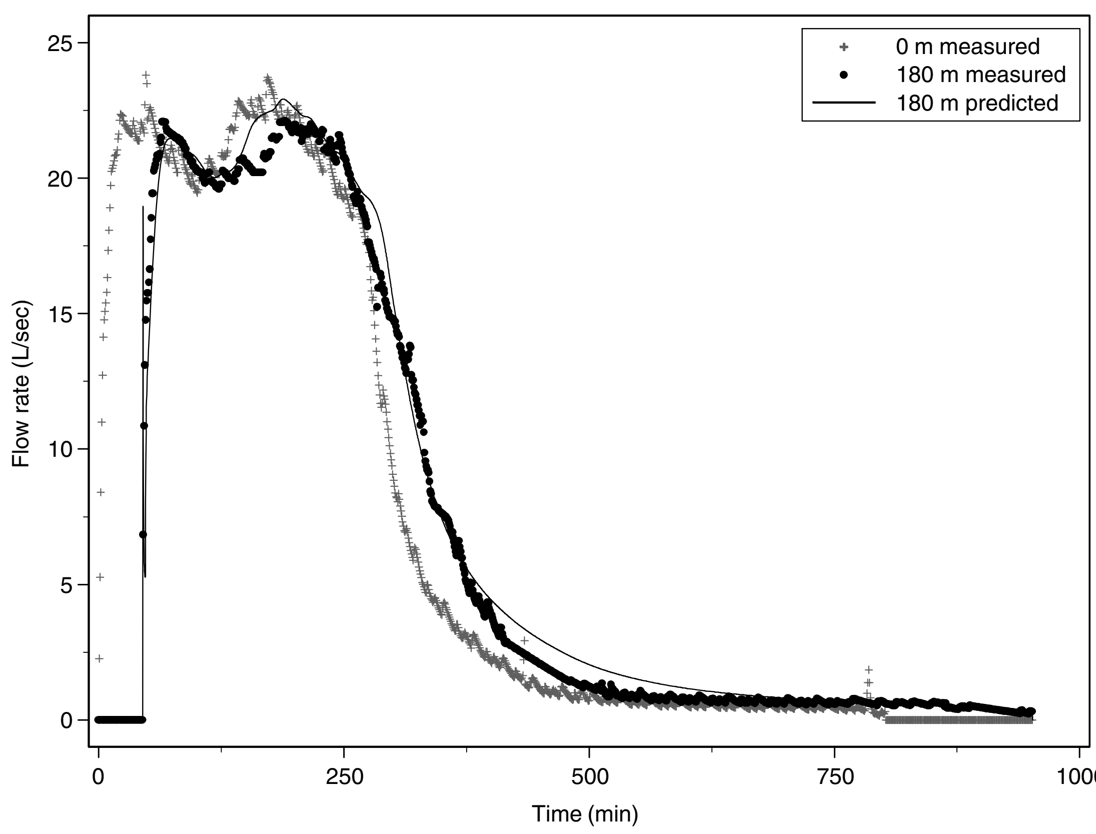

Fig. 3. Plot of measured and predicted flow rates in the farm drain for field investigation 1 (18-Dec-00) using a Manning's roughness of 0.011 and estimated infiltration coefficients.

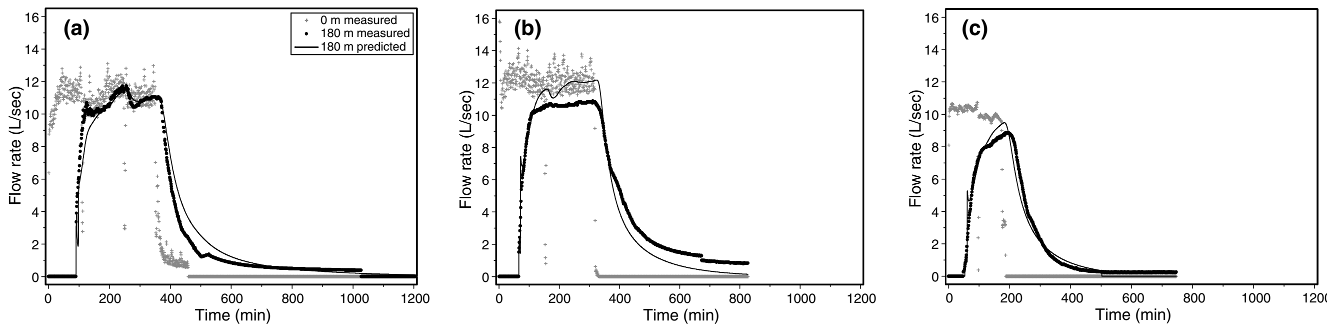

Fig. 4. Plot of measured and predicted flow rates in the farm drain for field investigations (a) 2:9-May-01, (b) 3:7-Jun-01, and (c) 4:31-Jul-01.

Table 2
Comparison of measured and predicted flow parameters for flow in the farm drain between Dec-00 and Jan-02

Field investigation	Date	Flow volume (ML)			Average flow rate (L/s)			
		Measured	Predicted	ε (%)	Measured	Predicted	s^2	COE[a]
1	18-Dec-2000	0.39	0.40	2.9	6.89	7.20	0.52	0.96
2	9-May-2001	0.23	0.23	0.3	3.35	3.50	0.39	0.99
3	7-Jun-2001	0.23	0.23	0.3	4.95	5.04	0.88	0.97
4	31-Jul-2001	0.10	0.10	0.2	3.74	3.90	0.41	0.98

[a] Nash and Suttcliffe Coefficient of Efficiency [40].

parameters were used. The modelled wetting front moved 180 m down the drain in 47 min, three minutes less than observed in the drain. At the end of the drain (180 m) flow volume was satisfactorily predicted ($\varepsilon = 3.1\%$) with 0.40 ML of flow predicted compared to the 0.39 ML measured. Flow rate was well predicted with a COE of 0.99, however over the flow period the average flow rate was over estimated, 7.4 L/s predicted compared to 6.9 L/s ($s^2 = 1.3$) measured.

The predictive capacity of the model was further investigated by adjusting the infiltration and Manning's roughness components of the model. The estimated infiltration rates used in the model accounted for less than 2% of the total volume of water, and while doubling the rate of infiltration marginally improved the prediction of flow volume ($\varepsilon = 2.9\%$) no significant change in the predicted flow rates (7.4 L/s) resulted. Manning's roughness coefficients between 0.01 and 0.03 [13] were used in Eq. (5), with the prediction errors minimised (s^2) when $n = 0.011$ (Fig. 3) and the model predicted an average flow rate of 7.2 L/s ($s^2 = 0.52$) and a total flow volume of 0.4 ML ($\varepsilon = 2.9\%$).

While optimising n reduced the variation between measured and predicted flow rates, the improvement in prediction was not sufficient to offset the benefits of being able to use literature values to approximate flow in the drain. To further investigate the ability of the estimated n values (Table 1), Eq. (5) was used to describe flow from field investigations 2, 3 and 4 (Fig. 4). Generally Eq. (5), combined with literature n values, provided an acceptable description of water movement in the irrigation drain. For all three field investigations, the total volume of flow was accurately predicted (Table 2) with ε less than 0.5%. While the prediction of average flow rates was acceptable (Table 2), the ability of the model to account for the variation in flow rates varied between the different events (Fig. 4), reflected in the mean squared error (Table 2). The residuals calculated for flow rate centred around zero, with residuals for investigations 2 and 4 normally distributed and the residuals for field investigation 3 arranged in two clusters either side of zero. This clustering of the residuals was not improved by changing the Manning's roughness or infiltration coefficients, suggesting that an unaccounted for variable affected the measured flow rates on the 7-Jun-01.

As the residual error associated with flow rates in all of the events was centred around zero with a COE greater than 0.95, and the total flow volumes were accurately predicted (Table 2), it was believed that the modified volume routing equation provided a sufficient basis for the subsequent modelling of P transfer in irrigation drains.

3. Modelling phosphorus in the irrigation drain

Phosphorus exported from irrigated pastures in south-eastern Australia is predominantly (>80%) in the <0.45 μm filtrate, operationally defined as dissolved [8,16,37]. Due to the dominance of dissolved P, this model focuses on the transport and interactions of the dissolved P fraction. Advection is the dominant transport process for dissolved P in a farm scale irrigation drain [6], which was modelled using a simple wave equation [52]:

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = 0, \tag{11}$$

where concentration (C), was a function of time (t), distance (x) and the water velocity (U). The partial derivatives in Eq. (11) were approximated by finite differences forwards in time and backwards in space, such that:

$$\frac{\partial C}{\partial t} \approx \frac{C_{(x,t+\Delta)} - C_{(x,t)}}{\Delta} \quad \text{and} \quad \frac{\partial C}{\partial x} \approx \frac{C_{(x,t)} - C_{(x-\delta,t)}}{\delta}. \tag{12}$$

Eq. (11) assumes that P behaved conservatively in the drain, with no interactions between P and the surrounding environment. Phosphorus however, does interact with the environment during transport in streams [21,27,34] and drains [7,9], with previous field investigations of P transport in farm drains suggesting that P uptake and release by bed sediments and P release by plant material were the dominant processes [7].

Phosphorus uptake and release during transport along the drain was modelled by incorporating the net change in concentration (λ) such that Eq. (11) becomes:

$$\frac{\partial C}{\partial t} + \mu\frac{\partial C}{\partial x} = \int_{x_0}^{x} \lambda(x')\,\mathrm{d}x', \tag{13}$$

with an explicit solution to the P transfer model obtained by combining Eqs. (12) and (13):

$$C_{(x,t+\Delta)} = C_{(x,t)} - \frac{\mu\Delta}{\delta}\left(C_{(x,t)} - C_{(x-\alpha,t)}\right) + \int_{x}^{x+\delta} \lambda(x')\,\mathrm{d}x'. \tag{14}$$

The net change in concentration (λ) was described by two rate equations (as developed below) chosen to describe P release by plants and P uptake and release by bed sediments.

3.1. Phosphorus release by plants

Research has shown that P release from plant material is initially rapid and decays over time to a constant rate [51]. This trend was described in the P model using a hyperbolic rate equation [23,49]:

$$\frac{\partial P_{\text{plant}}}{\partial t} = \frac{1}{a_{\text{plant}} + b_{\text{plant}}t}, \tag{15}$$

where P_{plant} describes P release per unit surface area of drain (mg/m^2) as a function of time using two empirical coefficients (a_{plant} and b_{plant}).

While the rate of P release is dependent on a number of factors including plant age, fertiliser history and rainfall intensity [49], there was insufficient information to incorporate these variables into the model. For simplicity it was assumed that P release from plants was a function of the mass of plant material and an estimate of the percentage P in the plants [35].

The mass of plant material was approximated by converting plant height into dry matter (dm) using a 'rule of thumb' for estimating pasture production [12], and correcting for estimated plant density. It was assumed that the pasture vegetation contained 0.3% P on a dry weight basis [44], with 70% of the P in the plant in a water soluble form [29]. Up to 87% soluble P can be leached from decaying vegetation over 96 h [10], with less P expected to be leached from live vegetation. For modelling purposes it was assumed that 40% and 70% was leached from live (live$_{\%}$) and decaying (decay$_{\%}$) vegetation, respectively, over this period.

Assuming that the coefficients were a function of the potential P leached over 96 h, a linear relationship was developed which assumed that a_{plant} equalled P leached in a 96 h period per unit area of drain, and that b_{plant} was linearly related by the equation:

$$\begin{aligned} a_{\text{plant}} &= 0.225\text{dm} \times 0.7\text{decay}_{\%} + 0.225\text{dm} \times 0.4\text{live}_{\%}, \\ b_{\text{plant}} &= 107 - 497a_{\text{plant}}. \end{aligned} \tag{16}$$

3.2. Phosphorus uptake and release by bed sediments

Phosphorus uptake and release by bed sediments is an equilibrium process [33] which is affected by a range of factors including the sorption capacity of the sediments, the total P content of the sediments [25] and the P concentration of the applied water [48]. Phosphorus uptake and release by sediments has previously been described using a range of equations including the Elovich equation [26,43].

To incorporate the effect of changing sediment characteristics and water quality over the 14-month modelling period a modified form of the Elovich equation (17) was used. The modified equation adjusts the magnitude of P uptake and release based on difference between the P concentration of water ($C_{(x,t)}$) and the equilibrium phosphate concentration (EPC) of the sediment [45]. The modified Elovich equation:

$$\frac{\partial P_{\text{bed}}}{\partial t} = \frac{-\exp(a_{\text{bed}})b_{\text{bed}}}{1 + \exp(a_{\text{bed}})t}\left(C_{(x,t)} - \text{EPC}\right)10, \tag{17}$$

describes P uptake or release per unit surface area of drain (P_{bed}) as a function of time since the start of flow, EPC (treated as an empirical parameter), $C_{(x,t)}$ and two empirical coefficients (a_{bed} and b_{bed}).

By combining Eqs. (14), (15) and (17) the model used to describe P transfer down a farm scale irrigation drain was obtained.

$$C_{(x,t+\Delta)} = \frac{U\Delta}{\delta}\left(C_{(x,t)} - C_{(x-\delta,t)}\right) + \frac{\partial P_{\text{plant}}}{\partial t}\frac{\Delta}{d} + \frac{\partial P_{\text{bed}}}{\partial t}\frac{\Delta}{d}. \tag{18}$$

3.3. Application of the phosphorus model

The P model (18) was applied to 14 months of flow data for a farm scale irrigation drain on the Macalister Research Farm. As noted in Section 2.1, management actions resulted in significant changes in drain condition, starting with a bare earth drain (Dec-00) progressing through a period of plant growth (May-01) plant decay post herbicide application (Jun-01 and Jul-01) and finally a second period of plant growth (Jan-02). Between Dec-00 and Jul-01, field investigations 1–4 were conducted [7] with flow measured at the top and bottom of the drain as previously described and water samples collected at the top of the drain using an ISCO model 3700 automatic sampler and the bottom of the drain using a vertically integrated sampler [39]. A fifth field investigation was conducted in Jan-02 when water samples were collected at the top and bottom of the drain as described, however there was no flow measured at the bottom of the drain for this investigation. The water samples were analysed for total dissolved P (0.45 μm filtrate), using molybdenum blue chemistry [36] on a Lachat Quickchem 8000 flow injection system (Zellweger Analytics Inc., USA).

The P model was initially applied to the five field investigations. Once the model had been optimised and relationships developed to describe the changes in parameters due to changing soil and plant conditions, the model was applied to the 13 irrigation and 9 rainfall runoff events (>0.10 ML) which occurred between Dec-00 and Jan-02. The model was then used to describe observed drain conditions (Scenario 1), bare earth drains (Scenarios 2 and 3), as well as an established grass drain (Scenario 4).

3.3.1. Input parameters

Application of the P model required estimates of input parameters, including coefficients to describe P release by plant material and P uptake and release by bed sediments. The P model also required flow velocity and depth of the water column (for each distance and time step), which were calculated from the output of the modified volume routing equation.

Estimates of plant height and density were used to describe P loss from plants and to provide parameter estimates for a_{plant} and b_{plant} (Table 3). Between the experimental flows linear interpolation was used to calculate plant height and density in the drain, as a gradual change in drain conditions was observed between experiments (but was not measured). Eq. (16) was then used to estimate a_{plant} and b_{plant} based on based on the estimates of plant height and density.

Estimates of EPC, a_{bed} and b_{bed} determined from laboratory data presented in [6], were initially used to describe P uptake and release by bed sediments in the drain, with values of $EPC = 0.16$, $a_{bed} = -9.7$ and $b_{bed} = 0.049$. However, treatment of sediments prior to laboratory investigations were likely to affect the validity of these coefficients when applied to the field situation [4], suggesting that field data may be required to optimise the model.

The model was then applied to the five field investigations, with EPC, a_{bed} and b_{bed} used to optimise the model in an iterative fashion. Unfortunately with the five field investigations having different sediment and plant characteristics as well as flow rates and concentrations (Fig. 2) there was limited data from which to uniquely parameterise the model. Due to the variation in conditions and the number of model parameters, the results from all five field investigations were used to optimise the P model, and develop relationships which were then used to estimate the parameters over the 14-month period. This approach meant that there were no independent events for testing. Ideally we would have had a larger number of data sets to enable some independent testing. We did undertake calibration using just 3 of the five sets, but the choice of the three sets affected the optimal parameter values. Some choices gave excellent test results (equivalent to when all five were used) while others gave poor test results.

The optimised parameters were then related to bed sediment properties that were measured prior to the experimental flows using linear regression, particularly the total P (TP) content of the sediment and P sorption characteristics (Table 4). The relationship between sediment properties and model parameters were then used to estimate the model input parameters for the 21 runoff

Table 3
Plant characteristics for the farm drain during the field investigations and estimated plant parameters used in the P model

		Field investigation				
		1	2	3	4	5
		18-Dec-00	9-May-01	7-Jun-01	31-Jul-01	31-Jan-02
Plant height	cm	0	15	10	10	10
Plant density	%	<5	85	80	70	50
Dead material	%	0	10	50	70	30
a_{plant}	–	–	0.20	0.39	0.26	0.13
b_{plant}	–	–	12.0	56.9	49.7	6.0

Table 4
Sediment characteristics for the farm drain measured immediately before the field investigations

		Field investigation				
		1	2	3	4	5
		18-Dec-00	9-May-01	7-Jun-01	31-Jul-01	31-Jan-02
pH	Water	7.8	7.8	7.7	7.6	7.1
EC	μS/cm	184	195	164	170	110
Total P	mg/kg	250	340	460	470	460
Langmuir P sorption maximum	mg/kg	1600	1600	1600	1500	1700
P sorption (10 mg/L)	mg/kg	770	720	700	710	680

events which occurred over the 14-month period on the basis of observed drain conditions and three alternate management strategies.

The assumptions used in determining the input parameters for the four scenarios varied. In Scenario 1 the TP content of the sediment and P sorption characteristics measured prior to experimental flows were extrapolated between events (Fig. 2) using linear interpolation. In Scenario 2 TP content of the sediment and P sorption characteristics were assumed to be constant over time. In Scenario 3 the TP content of the sediment and P sorption characteristics were adjusted over consecutive runoff events by using the change in P load between the top and bottom of the drain for each runoff event to alter P content of the sediment. In Scenario 4, it was assumed that the P content of bed sediments at the start of the 14-month period was equivalent to characteristics measured on 31-Jul-01 and changed in response to the change in P load between the top and bottom of the drain for each runoff event.

3.3.2. Statistical analysis

The performance of the P model (18) was investigated using the concentration and loads for the outlet of the drain over the five experimental flows. Three criteria were used to assess the model performance, (a) the relative error (ε) between the measured and predicted loads, (b) the mean squared error (s^2) of concentrations, and (c) a graphical investigation of the distribution of the residual values for concentration.

3.4. Phosphorus model—results and discussion

Phosphorus transfer down the farm scale irrigation drain during the first field investigation (18-Dec-00) was initially modelled using coefficients estimated from laboratory investigations. The modelled P concentrations observed at the end of the drain (180 m) were significantly higher than the measured values (Fig. 5a), with the P loads exported from the drain also over predicted (147 g measured, 267 g predicted). The results suggest that laboratory estimates of parameters were not sufficient to describe P interactions in a field situation. Despite the inability of the laboratory-based parameters to adequately describe P concentration changes in the field, the changes in concentrations observed suggest that the functional form of the equation was acceptable.

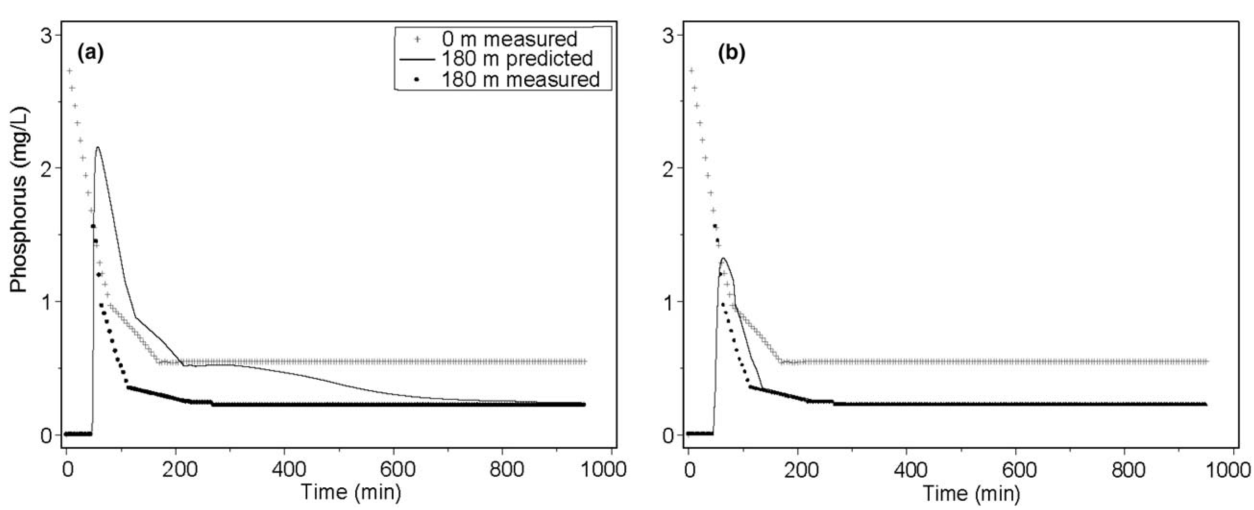

Fig. 5. Measured and predicted P concentrations versus time during the first field investigation (18-Dec-00), modelled using (a) laboratory based estimates of a_{bed}, b_{bed} and EPC, and (b) optimised using a_{bed}, b_{bed} and EPC.

To improve the fit between observed and predicted concentrations and loads the model was manually optimised by varying EPC, a_{bed} and b_{bed}, in an iterative fashion to minimise both ε and s^2. For the first field investigation (Fig. 5b), the optimised model predicted a P load of 157 g ($\varepsilon = 5.8\%$) and an average concentration of 0.39 mg/L ($s^2 = 0.018$). For the remaining field investigations the model was manually optimised by varying EPC, a_{bed} and b_{bed}, assuming that P release by plant material was adequately described using the estimated parameters (Table 3). Phosphorus load was predicted with ε less than 5% and s^2 was less than 5% of the average concentration (Fig. 6).

The coefficients for the optimised P transfer model (Table 5) were related to the total P content (TP of sediment), the P sorption capacity of the sediments (P sorption$_{(10mg/L)}$) and the inlet P concentration (C_0) using linear regression. These relationships were required to allow changing soil conditions over time to be used in determining the uptake and release of P by bed sediments in the model.

The additive model describing a_{bed} was significant ($p < 0.001$) and accounted for 64% of the variation in coefficient values.

$$a_{bed} = -3.8 - 0.021 \cdot (\text{TP of sediment}). \tag{19}$$

The additive model describing b_{bed} was significant ($p < 0.001$) and accounted for 96% of the variation in coefficient values.

$$b_{bed} = 0.41 - 0.00085 \cdot (\text{TP of sediment}) - 0.0024 \cdot \left(\frac{1}{C_0}\right). \tag{20}$$

The additive model describing EPC was also significant ($p < 0.001$) and accounted for 81% of the variation in coefficient values

$$\text{EPC} = 2.6 - 0.0031 \cdot (\text{Psorption}_{(10mg/L)}) + 0.022 \cdot C_0. \tag{21}$$

The results of the five field investigations were all used in developing the relationships between model parameters and soil characteristics, as the ‘information content’ of some of the data sets was not sufficient to uniquely define parameter values. While this meant that there were no independent data sets available to assess model performance, the confidence in the relationships was higher than if we had only used three experiments. When Eqs. (19)–(21) were used to determine model parameters for the five field investigations the relative error in the prediction of loads was less than 7%.

Using Eqs. (19)–(21), the coefficients describing P uptake and release by bed sediments could be varied over time in response to changing bed sediment characteristics. This allowed P transfer down the farm drain to be investigated over a 14-month period.

3.4.1. Scenario 1—Observed drain conditions

The first scenario investigated P transfer in the farm scale irrigation drain over a 14-month period in

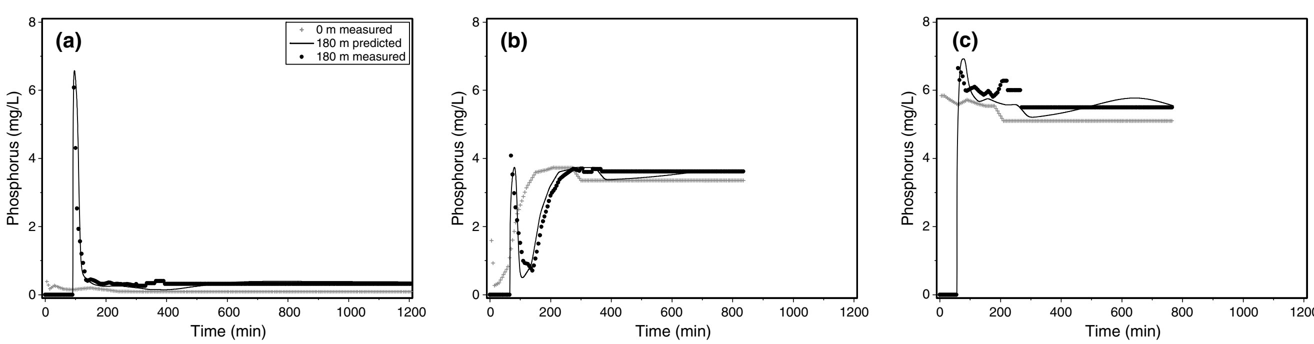

Fig. 6. Plot of measured and predicted phosphorus concentrations versus time in the ‘fenced’ drain on the Macalister Research Farm, using optimised coefficients for phosphorus uptake and release by bed sediments for experimental flows (a) 2:9-May-01, (b) 3:7-Jun-01, and (c) 4:31-Jul-01.

Table 5
Summary of phosphorus concentrations and loads predicted by the P model for field investigations in the farm drain, using the optimised coefficients for phosphorus uptake and release by bed sediments

Field investigation	Date	Optimised parameters			Phosphorus load			Average phosphorus concentration		
		a_{bed}	b_{bed}	EPC	Actual (180 m) (g)	Predicted (180 m) (g)	ε (%)	Actual (180 m) (mg/L)	Predicted (180 m) (mg/L)	s^2
1	18-Dec-2000	−8.5	0.198	0.20	148	156	5.75	0.37	0.39	0.018
2	9-May-2001	−11.6	0.105	0.32	108	103	4.57	0.47	0.44	0.021
3	7-Jun-2001	−13.2	0.0207	0.40	668	687	2.91	2.90	2.99	0.083
4	31-Jul-2001	−12.1	0.0008	0.50	633	620	2.08	5.94	5.81	0.137
5	31-Jan-02	−14.5	0.0310	0.50	776	808	4.16	0.93	0.97	0.405

response to observed drain conditions (Tables 3 and 4). As previously discussed plant and sediment characteristics were only measured prior to field investigations, this meant that drain conditions between field investigations were estimated using linear interpolation, and Eqs. (16) and (19)–(21) were used to calculate model parameters.

Consistent with the first field investigation, the model results suggest that P uptake by bed sediments was the dominant process while the mass of plant material in the drain was small, with a significant decrease in P concentration and loads with distance down the drain (i.e., P uptake by sediments > P release by plants). The model predicted net uptake by bed sediments over the first 10-runoff events, which resulted in the reduction of P loads by 27%. Over the 10 runoff events, 7.8 kg P was sorbed in the drain (12.4 g/m^2 of sediment surface), and the total P content of the sediment increased from 250 to 340 mg/kg, suggesting that the sorbed P was stored to a depth of approximately 90 mm.

When plant material started to build up in the drain, predicted P loads either increased or decreased depending on the net balance between P release by plants and P uptake by bed sediments. For example, on the 27-Sep-01 the P load predicted by the model increased over the length of the drain (5567–5712 g), while at other times including an event on the 15-Jun-01 the P load decreased (477–414 g).

Over the 14 month period the results suggest that despite P sorption by bed sediments in the first 10 runoff events, the P transported down the drain over the whole period was reduced by 9%. While optimised coefficients generally predicted P transfer with a ε less than 5%, there was no independent data to validate the optimised model against. Therefore P loads in the drain were reduced by somewhere between 0% and 17%, assuming a 10% prediction error.

Phosphorus uptake by bed sediments was greatest in the bare earth drain, prior to the accumulation of live and decaying vegetation. This suggests that the greatest potential for drain management to significantly reduce farm scale P export lies in the maintenance of a bare earth drain. Scenarios 2 and 3 both investigate the effect of maintaining a bare earth drain over the 14-month period.

3.4.2. Scenario 2—Bare earth with no change in sorption potential

Scenario 2 was designed to investigate the maximum potential of a bare earth drain to reduce P export, based on the premise that P sorbed in the previous event did not accumulate in the soil. This meant that the total P content of the soil, and P sorption capacity of the soil were constant over time, which meant that a_{bed} was constant, while b_{bed} and EPC varied in response to the inlet P concentrations (Eqs. (20) and (21)). While, the assumption that P does not accumulate in the soil is unrealistic, it provides an idea of the maximum possible benefit that may be obtained from maintaining a bare earth drain.

With the maximum P sorption capacity of the sediment maintained over the 14-month period, the model suggested that net P uptake occurred in all events with the exception of one runoff event where low P concentration channel water was pumped directly into the drain. Over the 14-month period the model predicted a 28% decrease between loads entering the drain and predicted loads at the bottom of the drain. However, a more realistic scenario, would allow the sorbed P to build up in the surface sediments over time, reducing the P sorption capacity over consecutive runoff events.

3.4.3. Scenario 3—Bare earth with an increase in soil TP over time

Scenario 3 was designed to present a more realistic indication of the effect of maintaining a bare earth drain on P uptake by bed sediments. The model was applied in a sequential nature with the P sorbed by the sediment in the previous event, incorporated into the top 100 mm of the sediment, based on the assumption that over time diffusion of P in the soil solution and infiltration would leach P beyond the top 10 mm generally believed to interact with water [1]. The sorbed P was used to calculate new sediment P characteristics and using Eqs. (19)–(21) model parameters for the next flow event. The assumption of a 100 mm storage depth was also consistent with the predicted P uptake by sediments and measured change in sediment TP observed during the first 10 flow events in Scenario 1.

Over the 14 month period the model predicted a total P increase in the sediments from 250–386 mg/kg, while P $\text{sorption}_{(10\text{mg/L})}$ decreased from 770 to 700 mg/kg. Despite the accumulation of P in the surface soils P uptake by bed sediment still reduced P loads in the drain by 19% (11–27%). Predicted P loads at the bottom of the drain in Scenarios 2 and 3 diverged over time (Fig. 7), as the P sorption capacity of the sediment decreased.

Phosphorus uptake by the drain sediments would be further reduced if the depth of sediment in which the sorbed P was stored decreased. For example, with a storage depth of 10 mm, the TP content of the sediment had increased to 471 mg/kg after 10 flow events, while P $\text{sorption}_{(10\text{mg/L})}$ had decreased from 770 to 660 mg/kg. Over the first 10 flow events, predicted P sorbed by the sediments in the drain decreased significantly when the sorbed P was stored in the top 10 mm (2302 g) compared to the top 100 mm (6797 g) of sediment.

The results from Scenarios 2 and 3 highlight the finite capacity of sediment to sorb P. Assuming that a bare earth drain could be maintained indefinitely, sediment in the drain would reach a level of P saturation where no net uptake or release would be expected over an irri-

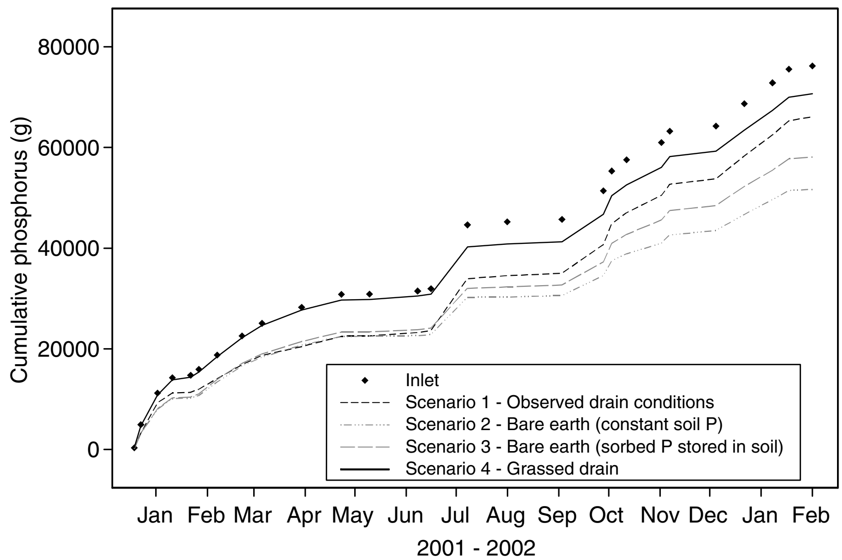

Fig. 7. Cumulative P loads estimated for the end of drain over 14-month period using the four different management scenarios.

gation season. The P uptake capacity of a bare earth drain may be maintained over an extended period by removing the surface sediment in the drain at regular intervals, with sediment and P returned to the pastures. Lining the drain with modified adsorbents such as Phoslock™ [46], or Fe-enriched materials could also increase or maintain the P uptake capacity of a bare earthen drain over time. There is however a risk associated with using sediments or chemical adsorbents in a bare earth drain to reduce P loads, in that during high flow events, sorbed P may be scoured from the drain and transported to our inland water systems if the drainage system is not engineered properly.

3.4.4. Scenario 4—An established grass drain, with no management

The final scenario, assumed that the drain was not cleared with an excavator at the start of the experimental period, and there was no herbicide application or grazing to remove plant cover. Scenario 4 was based on an assumption of 100 mm plant height, 80% ground cover and 20% decaying plant material which remained constant overtime. Assuming that the drain was not cleared at the start of the experimental period, initial P content of the sediment was assumed to be 460 mg/kg with changes in P load predicted between the top and bottom of the drain used to adjust sediment characteristics over the modelling period.

Phosphorus transfer modelled using these simplistic assumptions of plant growth and sediment properties, resulted in a 2% decrease in P loads in the drain (8% increase–11% decrease). There was little variation in the level of P uptake or release over time, suggesting that an established pasture based drain had little net effect on P transport.

4. Concluding discussion

The flow and P transfer models presented here adequately describe water and P movement down a 180 m long farm scale irrigation drain. Flow in the drain was well predicted by the model, with less than 5% error between measured and predicted values when literature values for Manning's roughness and estimates of infiltration were used. Phosphorus loads were assumed to have an error of 10% after optimisation, as there was no independent data to validate the results.

In the experimental drain on the Macalister Research Farm, the model predicted reductions in P loads of 9%, 28%, 19% and 2% for Scenarios 1–4, respectively. In Scenario 1, observed conditions in the irrigation drain (bare earth, plant growth, herbicide application, senescence and plant growth) suggested that clearing out the drain using an excavator every 12–18 months could reduce P export from the drain by approximately 10%. In Scenarios 2 and 3, maintenance of a bare earth drain appeared to reduce P loads exported from the drain catchment area. When the sorbed P did not accumulate in the bed sediments the model predicted a 28% reduction in P loads over the 14-month period. However, assuming that the sorbed P was stored in the top 100 mm of drain sediments, the sorption capacity of the sediments declined over time, with P loads over the 14-months reduced by 19%. Further modelling results showed that if P was stored in the top 10 mm of the drain, the sorption capacity of the sediments decreased at a significantly greater rate and P uptake by bed sediments was further reduced. The established grassed drain in Scenario 4 had minimal effect on P loads during transport, with a 2% change in P loads between the top and bottom of the drain over the 14-month period.

While, the model was only used to describe P transfer in a single farm scale irrigation drain, the results may be extrapolated to other drains and other irrigation regions, where

- increasing inflow into the drain per unit width would reduce the net change in P loads due to increased flow depth and/or increased water velocity and therefore reduce reaction time per unit volume of water,
- increasing the slope of the bed sediments would increase the rate of water movement and therefore reduce reaction time per unit volume of water,
- increased infiltration would decrease the loads of P transported down the drain and reduce the rate at which P accumulated in the surface soils through movement of P through the soil profile,
- different P sorption capacities, and total P content of drain sediments would affect the ability of the sediment to take-up and release P.

This study suggests that a farm scale irrigation drain can significantly reduce P export between the paddock and the farm boundary, with the maintenance of a bare earth drain, or regularly clearing of the drain with an excavator every 12–18 months likely to reduce P export from the drain and therefore the farm. Assuming that the risk of scouring these bed sediments from the farm to streams is low, maintaining bare earth drains would appear to be a feasible strategy to incorporate into nutrient plans at the farm scale. However, the recurring expense of herbicide application and clearing drains with an excavator, suggest that drain management for P retention is unlikely to be a cost-effective or widely adopted nutrient reduction strategy, given that the potential reduction in P export is in the order of only 10%.

Even though the results from this investigation are unlikely to inspire practice change on the farm, it provides a good understanding of P transfer between the paddock and the farm boundary. The P transfer model has the potential to be used to investigate and evaluate alternate management strategies across a range of drain conditions without costly long-term field investigations.

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Appendix A. Solution for the volume routing equation

The modified volume routing equation (2), notation summarised in Table 6, was solved using a relatively simple advective finite difference scheme, which assumes that the equation has an advective nature. Eq. (2) may be presented in a form where k is a function of $\mathrm{d} = \partial v/\partial x$ such that,

$$\frac{\partial v}{\partial t} + k\left(\frac{\partial v}{\partial x}\right)\sqrt{S - \frac{\partial^2 v}{\partial x^2}} = \int_{x_0}^{x} i(x')\,\mathrm{d}x', \tag{22}$$

small disturbances about a uniform steady flow of depth d_0 and flow per unit width q_0 were considered when inflow was zero (i.e., $i = 0$):

$$v = d_0 x - q_0 t + v_1 \tag{23}$$

and v_1 is a small deviation from the uniform value. This can be substituted into Eq. (22),

$$-q_0 + \frac{\partial v_1}{\partial t} + k\left(d_0 + \frac{\partial v_1}{\partial x}\right)\sqrt{S - \frac{\partial^2 v_1}{\partial x^2}} = 0. \tag{24}$$

The terms in Eq. (24) were expanded using both Taylor series

$$k\left(d_0 + \frac{\partial v_1}{\partial x}\right) = k(d_0) + \frac{\partial v_1}{\partial x}\frac{\partial k}{\partial d}(d_0) + \cdots \tag{25}$$

and Power series

$$\sqrt{S - \frac{\partial^2 v_1}{\partial x^2}} = \sqrt{S}\sqrt{1 - \frac{1}{s}\frac{\partial^2 v_1}{\partial x^2}} \approx \sqrt{S}\left(1 - \frac{1}{2S}\frac{\partial^2 v_1}{\partial x^2} + \cdots\right). \tag{26}$$

The approximations obtained from the series were substituted into Eq. (24):

$$-q_0 + \frac{\partial v_1}{\partial t} + \left(k(d_0) + \frac{\partial v_1}{\partial x}\frac{\partial k}{\partial d}(d_0)\right)\sqrt{S}\left(1 - \frac{1}{2S}\frac{\partial^2 v_1}{\partial x^2} + \cdots\right) = 0. \tag{27}$$

Eq. (27) was then multiplied out. Any terms which were the product of v_1 or its derivatives were removed from the solution as they are a function of a small deviation. Leaving the equation:

$$-q_0 + k(d_0)\sqrt{S} + \frac{\partial v_1}{\partial t} - k(d_0)\frac{\sqrt{S}}{2S}\frac{\partial^2 v_1}{\partial x^2} + \sqrt{S}\frac{\partial k}{\partial d}(d_0)\frac{\partial v_1}{\partial x} = 0. \tag{28}$$

Uniform flow law states that $q_0 = k(d_0)\sqrt{S}$ allowing the first terms in Eq. (28) cancel out, leaving the equation for v_1:

Table 6
Summary of notation used in the model

Symbol	Description	Unit
ε	Relative error between measured and predicted flow and P loads	–
a_{bed} and b_{bed}	Empirical coefficients	–
a_{plant} and b_{plant}	Empirical coefficients	–
a_{inf} and r_{inf}	Empirical coefficients in the infiltration equation	–
B	Width of channel or drain	m
C	Phosphorus concentration	mg/L or g/m^3
d	Depth of water in the drain	m
EPC	Equilibrium phosphorus concentration	mg/L or g/m^3
I	Inflow per unit width of drain	m^2/s
K	Conveyance	m^3/s
k	Conveyance per unit width of drain	m^2/s
n	Manning's roughness coefficient	s/m$^{1/3}$
P_{bed}	Uptake or release of phosphorus by bed sediments	g/m^2
P_{plant}	Release of phosphorus by plant material	g/m^2
q	Inflow	m^3/s
c_k	Constant	–
S	Slope of the bed sediment	–
s^2	Mean squared error	–
t	Time	s
U	Velocity	m/s
V	Volume of water	m^3
v	Volume of water per unit width of drain	m^2
x	Distance	m
Z	Cumulative infiltration	m
λ	Concentration change due to uptake or release of phosphorus by plants and bed sediments	g/m^3
Δ	Time step used in model	s
δ	Distance step used in model	m

$$\frac{\partial v_1}{\partial t} + \underbrace{\sqrt{S}\frac{\partial k}{\partial d}(d_0)}_{\substack{\text{kinematic wave speed}\\\text{(advection)}}} \frac{\partial v_1}{\partial x} = \underbrace{\frac{k(d_0)}{2\sqrt{S}}}_{\text{diffusion}} \frac{\partial^2 v_1}{\partial x^2}. \tag{29}$$

Numerical scheme

Assumes that inflow is zero, the linear advection term in Eq. (29) is used in the development of a numerical solution to the modified volume routing equation. The linear advection term from Eq. (29) was added into the modified volume routing equation (22), taking a term to the right hand side:

$$\frac{\partial v}{\partial t} + \sqrt{S}\frac{\partial k}{\partial d}(\partial v/\partial x)\frac{\partial v}{\partial x} = \sqrt{S}\frac{\partial k}{\partial d}(\partial v/\partial x)\frac{\partial v}{\partial x} - k(\partial v/\partial x)\sqrt{S - \frac{\partial^2 v}{\partial x^2}} \tag{30}$$

so that the right hand side looks like a diffusion term if we were to linearise it. An advection solution was introduced where

$$v_{(x,t+\Delta)} \approx v_{(x-\varpi\Delta,t)} \quad \text{and} \quad \varpi = \sqrt{S}\frac{\partial k}{\partial d}(\partial v/\partial x).$$

This advection scheme states that 'what was here now was upstream one time step ago (Δ) at a distance equal to that traversed by the stream in that time step'. This solution has better numerical properties than conventional finite difference schemes [14].

The volume at $(x - c_k\Delta)$ was approximated using a Taylor expansion where

$$v_{(x-c_k\Delta,t)} \approx v_{(x,t)} - c_k\Delta\frac{\partial v}{\partial x} + \frac{(c_k\Delta)^2}{2}\frac{\partial^2 v}{\partial x^2} - \cdots \tag{31}$$

If we represent the right hand side of Eq. (30) by N and add $N\Delta$ onto the approximate solution to the advection scheme presented in (31), we get

$$\begin{aligned} v_{(x,t+\Delta)} &\approx v_{(x-c_k\Delta,t)} + N\Delta \\ &\approx v_{(x,t)} - c_k\Delta\frac{\partial v}{\partial x} + \frac{(c_k\Delta)^2}{2}\frac{\partial^2 v}{\partial x^2} + N\Delta, \\ v_{(x,t+\Delta)} &\approx v_{(x,t)} - c_k\Delta\frac{\partial v}{\partial x} + \frac{(c_k\Delta)^2}{2}\frac{\partial^2 v}{\partial x^2} \\ &\quad + \Delta\left(c_k\frac{\partial v}{\partial x} - k(\partial v/\partial x)\sqrt{S - \frac{\partial^2 v}{\partial x^2}}\right), \\ v_{(x,t+\Delta)} &\approx v_{(x,t)} + \frac{(c_k\Delta)^2}{2}\frac{\partial^2 v}{\partial x^2} + \Delta k(\partial v/\partial x)\sqrt{S - \frac{\partial^2 v}{\partial x^2}}. \end{aligned} \tag{32}$$

Reintroducing the infiltration component into Eq. (32) we get a numerical scheme where t_a represents time as a function of distance down the drain as infiltration only begins once flow has started.

$$v_{(x,t+\Delta)} \approx v_{(x,t)} + \frac{(c_k\Delta)^2}{2}\frac{\partial^2 v}{\partial x^2} - \Delta k(\partial v/\partial x)\sqrt{S - \frac{\partial^2 v}{\partial x^2}} - \int_{x_0}^{x}\frac{\partial Z}{\partial t_a}\delta'. \quad (33)$$

In the application of the equation, a three point centred difference formula was used to estimate the derivatives such that

$$\frac{\partial v}{\partial x} \approx \frac{v_{(x+\delta,t)} - v_{(x-\delta,t)}}{2\delta},$$

$$\frac{\partial^2 v}{\partial x^2} \approx \frac{v_{(x+\delta,t)} + v_{(x-\delta,t)} - 2v_{(x,t)}}{\delta^2}$$

and

$$\frac{\partial Z}{\partial t} \approx -a_{\text{inf}}(t_a^{r_{\text{inf}}} - (t_a + \Delta)^{r_{\text{inf}}}).$$

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