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On gate stroking and upstream and downstream control

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Abstract

The problem of computing upstream gate motions to bring about desired downstream flows at a regulator is considered. We introduce a method based on linear systems theory that can use an accurate direct numerical simulation program to solve the problem. However, results show that the very idea of computing all the details of the upstream gate motions is flawed and that it is more reasonable just to use the upstream gate motions to satisfy approximately the downstream flow requirements. The detailed satisfaction of those requirements is done better by implementing control measures at the regulator concerned.

INTRODUCTION

The concept of gate stroking was introduced by Wylie (1969), with the aim of calculating the movements of an upstream gate in a canal so as to bring about a desired variation in discharge at a downstream gate with the aim of minimising surface disturbances. In more recent years the concept has often been described more as one of "feedforward" control, where control measures are applied in anticipation of an expected event in order, for example, to avoid unacceptable changes in downstream water levels. The movements of the upstream gate are designed so as to supply as much as possible the desired flow history at the downstream gate with a minimum of operation of the downstream gate and subsequent wave generation.

There have been many papers devoted to the subject since 1969, and although much has been claimed for the variety of methods developed, many of the results obtained have been unsatisfactory. Gate stroking as such seems to have been applied little in practice, although the American Society of Civil Engineers Task Committee on Canal Automation Algorithms has given some emphasis to feedforward control.

As channel systems are progressively moving to real-time control, the opportunity now exists to take advantage of modelling and system identification techniques employed by the wider process control industries. Rubicon Systems Australia is developing such control systems that are based on observed data and are less computationally demanding than physical models. This development is using "grey box" system identification where prior physical knowledge is incorporated into physical models. This

approach makes use of the best features of physical modelling and system identification in order to achieve optimal channel control. As demand forecasting systems also become integrated with control systems, channel operations are not only taking advantage of pre-emptive distant upstream or downstream actions, but can also rely on future control strategies based on demand predictions. Rubicon is also extending the technique of system identification into the area of demand prediction as an integrated channel control technology.

In this paper we examine the equations governing the motion of waves and flows in canals and we show that the technique for gate stroking proposed by all authors, of stepping backwards in space for all times of the motion desired to control, is in fact computationally defective. It is equivalent to computing with negative friction such that any disturbance, however slight, will in many cases invariably require huge and unreasonable upstream gate motions. We present a new means of computing the problem, which can make use of existing software without any modification to solve the reverse problem. However, in general we warn against the detailed computation of gate stroking and suggest that more heuristic measures are adequate and justified, including the aid of established practice, the use of accurate numerical simulation, and most importantly, control measures at the downstream gate.

LITERATURE SURVEY

Wylie developed the approach on which all previous methods for gate stroking have been based. Starting from a specified time history at the downstream gate for both surface elevation and discharge, the equations are solved numerically backwards in space along the channel, finally yielding the time history required at the upstream gate. He used the method of characteristics and reported no problems. Bodley and Wylie (1978) extended Wylie's ideas on gate stroking to a system of three pools.

Eli *et al.* (1974) used an implicit finite difference method instead of the method of characteristics. They found some good agreement. However, they noted that (p599) "some problems were encountered, particularly at low flows. When the actual discharge was routed backward the program failed to provide stable results."

Cunge et al. (1980) have been the only writers to have criticised the very concept of computing gate stroking by proceeding backwards in time and space. They observed: "Such a calculation would have to proceed backwards in time, which would be feasible if there were no energy losses. But energy losses must of course be included, and in this case the so-called inverse calculation becomes virtually impossible. Physically speaking, local perturbations in the flow are smoothed out as time advances due to friction effects; a forward-in-time calculation models this behaviour. But to proceed backward in time implies the reconstruction of the particular perturbations which show up as smoothed flow features at a later time." They go on to write: "Indeed, one would often like to be able to prescribe the opening of upstream gates in such a way as to obtain the desired downstream hydrograph. Such a problem is, mathematically speaking, an 'inverse' problem and as such is ill-posed, at least for problems of the hyperbolic type. Nevertheless some researchers have attempted to solve this inverse problem for practical applications (Bodley and Wylie, 1978). It is possible to obtain reasonable upstream flow variables (stage and discharge hydrographs) using downstream ones if the following conditions are satisfied: friction must be very small (negligible); non-linearity of the convective velocity term must not be strong; the distance between the regulators must be small. Even then the only method which can be used is the method of characteristics with all its complexity and unwieldiness. In order to define upstream releases it is better to run several test cases using a model built with the aid of a standard, efficient modelling system instead of a specially written inverse problem program. The celerities thus obtained for different volumes stored within the system of canals and for different discharges will be a sufficient guide for the planning of the release, whose propagation along the channel can then be checked using the same model." They presented those comments without quantification. We will attempt to do that below, but we will show that their qualitative comments are quite correct.

Falvey (1987) provided few details, but he noted (p177): "The gate motions determined by this method can be quite irregular if the flow changes are large". His studies also showed a need for additional check gates since some pools were too long for adequate control of the transients. He advocated a hybrid system of control, where for "discharge variations which are small relative to the capacity of the canal, the canal would be controlled with a local control algorithm ... large flow changes would be accomplished using gate stroking concepts".

Liu *et al.* (1992) solved the problem by a scheme that was "explicit and numerically stable". They observed large spikes in the input necessary, but did not explain why they occurred. They noted that "Since the implicit schemes have the advantage of being unconditionally stable numerically, they are now applied more widely than the explicit schemes." Interestingly they wrote that (p683): "... oscillations caused by smaller time interval do not indicate the instability of the Backward-Operation method but its superior accuracy". They found that "the oscillations in the computed upstream discharge hydrograph are amplified with the distance from the downstream section. However, the oscillations cause the failure of the computation".

Szymkiewicz (1993) used an implicit four-point scheme. He computed disturbances in a pool 49km long, and observed that "These data show (that not all) remarks of Cunge *et al.* on the limitations of solution of the inverse flow routing are reasonable", and went on to note "... the applied solution schemes are dissipative, and this also causes smoothing. Dissipation is a disadvantage of the numerical scheme, but ensures a stable solution of the hyperbolic nonlinear equations." However later (p118) "It can happen that for unrealistic imposed functions (of elevation and discharge at the downstream end) the corresponding functions (at the upstream end) do not exist. For instance, it is not possible to require the hydrograph to have a sharp form at the end of the long river reach of large roughness. It is well known that under such conditions the hydrograph is strongly smoothed at a downstream end because of friction."

In 1995 the American Society of Civil Engineers (ASCE) held a specialty conference at which several papers on the automatic control of canals were presented. One of the most relevant to this work was Bautista *et al.* (1995). They found that implicit methods perform better than explicit, but that when they computed forward again that the results are "smeared out". This would arise from numerical diffusion in both directions. They presented results showing the methods in all sorts of trouble. The explicit method performed particularly poorly. This may well be because it has less numerical diffusion. Some computational difficulties were experienced also with an implicit linearised model: "A loss of numerical accuracy with the linearised equations was identified as the source of the problem. New calculations were performed with smaller space and time steps but round-off errors destroyed the solution before adequate results were obtained." They noted that "... solutions computed with the implicit model reproduce the demand outflow hydrographs with great accuracy, when the prescribed changes were gradual. ... Better results can be expected with (the US Bureau of Reclamation) Gate Stroking Model with extreme transients, but in such cases neither model is entirely accurate and the solution may be altogether impractical ... with negative inflows required."

Burt *et al.* (1995) examined the influence of canal pool properties on the speed with which a canal could respond to unanticipated downstream withdrawals. This last study essentially determined the limitations on the ability of feedback control systems to achieve downstream control. From them, one can quantify the amount of flow change that can be accommodated by feedback alone. Greater flow changes require advance knowledge and feedforward routing, *i.e.* control measures applied in anticipation of an expected event in order, for example, to avoid unacceptable changes in downstream water levels. These studies were independent of gate hydraulics and control-algorithm characteristics.

In 1998 the ASCE Task Committee on Canal Automation Algorithms produced a report in the form of several papers. Strelkoff *et al.* (1998) provided a lot of evidence supporting the use of some sort of gate stroking. They wrote "Over the past several decades much attention has been given to methods for (1) controlling canal downstream water levels or volume with feedback control; (2) routing flow changes through canals with open-loop or feedforward control; and (3) utilizing local structures for

controlling either water levels or flows. However the success of any of these schemes is largely dependent upon the properties and characteristics of the canal itself, independent of the control method being used. However there is little in the literature examining the limitations that canal properties place on controllability."

Finally, at the end of the conclusion they observe "Our analyses suggest that not all flow changes in a canal pool can be accommodated by feedback alone. The amount of flow change that can be handled just by feedback is dependent upon the pool properties, the amount of allowable depth or pool volume change, and the properties of the feedback controller. This result emphasizes the need to include both feedback and feedforward components into canal control systems."

We will see below that all these reported phenomena can be explained as being simply due to friction in the canal. If the equations are solved backwards in space and time, then friction acts in a reverse sense and there is a marked tendency for all computational methods to blow up, unless they have an inbuilt numerical inaccuracy which leads to extra diffusion which causes artificial stability. Here we examine the application of a new method to solve a particular gate stroking problem. However it does not solve the problems of gate stroking, as we go on to identify that the problem is inherently full of difficulties, much as stated by Cunge *et al* (1980).

THEORY FOR WAVES IN CANALS

The flow of water and the propagation of waves in canals are described well by the St Venant long wave equations. Here we present them in the form where the dependent variables are the cross-sectional area A and discharge Q:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0,\tag{1}$$

$$\frac{\partial Q}{\partial t} + \left(\frac{gA}{B} - \frac{Q^2}{A^2}\right)\frac{\partial A}{\partial x} + \frac{2Q}{A}\frac{\partial Q}{\partial x} + gAS_f - gAS_0 = 0, \qquad (2)$$

where x is the distance along the waterway, t is time, g is gravitational acceleration, B is the width of the surface, S_f is the friction slope, and S_0 is the bed slope. Usually S_f is approximated by Manning's law $S_f = n^2 Q^2 P^{4/3} / A^{10/3}$, where *n* is Manning's coefficient and *P* is the wetted perimeter of the cross-section. It is possible to recast these two partial differential equations as four ordinary differential equations. In the so-called Characteristic Formulation it is possible to deduce that information (in the form of the gradient of the characteristics) proceeds up and down the canal at a speed given by $\sqrt{gA/B}$, which can be interpreted as $\sqrt{g \times \text{Mean depth}}$. The traditional interpretation of the St Venant equations is that it is the speed with which waves actually travel up and down the canal. While this is true if there is no friction, in general this is not correct. Also, the practical interpretation of the behaviour of waves in canals is that they travel without a great deal of diminution due to friction. In fact, for practical values of channel quantities, waves in a channel may be markedly diffused so that they arrive downstream considerably diminished in height and spread out much more in space and time. Insight into this process can be had if we consider approximations to equations (1) and (2). For small variations of flow and area about values Q_0 and A_0 , corresponding to uniform flow down the channel, after some mathematical operations, making assumptions that friction is the dominant effect we find that we can represent conditions in the channel by the single equation

$$\frac{\partial Q}{\partial t} + (U_0 + V_0) \frac{\partial Q}{\partial x} = \kappa \frac{\partial^2 Q}{\partial x^2}, \qquad (3)$$

with a similar equation for A, where we now use Q and A for discharge and area relative to the base flow in the canal. The equation is of advection-diffusion type, where $U_0 = Q_0 / A_0$ is the mean velocity along the channel, and where

$$V_0 = \frac{2}{3} U_0 \left(1 - \frac{A_0}{P_0} \frac{dP_0}{dA_0} \right), \tag{4}$$

where P_0 is the wetted perimeter corresponding to the reference flow of area A_0 . The coefficient κ is given by

$$\kappa = \frac{U_0}{2gS_0} \left(\frac{gA_0}{B_0} - {V_0}^2 \right).$$
(5)

Equation (3) is the basis of advection-diffusion flood routing, and is widely used in hydrology. It shows more clearly the nature of wave propagation in a canal, and it is quite different from the usual interpretations based on the St Venant equations (1) and (2), which ignore the effects of friction. Solutions are waves that propagate downstream at a velocity $U_0 + V_0$. In equation (4), for wide channels the term involving the derivative of perimeter is small, such that $V_0 \approx 2/3 U_0$ and $U_0 + V_0 \approx 5/3 U_0$. This means that waves in channels with friction actually propagate at a speed of about 5/3 times the speed of the water in the canal, usually rather smaller than the value of $\sqrt{g} \times \text{Mean depth}$, the widely-accepted value. The term on the right of equation (3) involving the wave which arrives at the downstream end is considerably lower and longer than that which entered upstream.

A NEW METHOD FOR GATE STROKING

In previous work on gate stroking suites of programs have been written which solve the St Venant equations backwards in space along the channel, given the complete desired flow and surface history at the downstream end. As noted above there have been many difficulties encountered. Here we attempt a different approach to the work, whereby we can use existing programs that solve the equations in the usual manner, forward in time along the channel. A simple modification based on linear systems theory, then not only gives us a method of simply performing flow routing computations without having to invoke the full solution techniques, but gives us a method which can solve the gate stroking problem at the same time. This method also demonstrates graphically how the latter is an operation that should often not be attempted.

Fundamental ideas from linear systems theory have been extensively used in hydrology, for example in the form of the unit hydrograph (Chow *et al.*, 1988). Dooge (1973) and Keefer (1976) amongst others have used this approach in the study of flow in channels. Napiórkowski (1992) presented a summary article of several papers of Dooge and Napiórkowski and Strupczewski. The output from a system, in this case the outflow at the downstream regulator, can be written in terms of the inflow at the other end, expressing it at a point in time as a weighted integral of the input. The weight function in that integral is the transfer or system function which expresses the effect downstream of a single unit of flow increase at the upstream end. Here we write it in discrete form, replacing the integrals by sums. There are J values of inflow to the channel at the upstream end, denoted by I_j for j = 0, 1, ..., J - 1. We suppose that the transfer function of the system is the sequence of K numbers h_k

for k = 0, 1, ..., K - 1, such that if the input to the system were a single spike of flow at time 0, then the values of the h_k would be the resultant outflow hydrograph for times t_k . Then, at time step n, O_n is the output at the downstream end that we write as a convolution sum

$$O_n = \sum_{j=0}^{n \le N} I_j h_{n-j}, \text{ for } n = 0, 1, \dots, N-1.$$
(6)

This expresses the fact that the flow at time step n is simply that due to the sum of each of the flows which entered the system at step j multiplied by the influence function for the difference in time measured by n - j. As the inflow sequence is J points long and the effect of the inflow lasts for K points, the total length of the outflow sequence is N = J + K. Strictly speaking, this approach is for systems which are linear, where all changes and inputs to the system may be linearly combined, such as the advection-diffusion equation (3), and not as in the St Venant equations where products and nonlinear functions of the flow variables Q and A occur. In many irrigation applications, however, changes about a base state of flow are quite small, and the system is very nearly linear.

In the usual situation where one knows the inflows and outflows and wants to obtain the transfer function so that subsequent outflows can be computed simply for arbitrary inflows, the procedure is to write equation (6) as the matrix equation

$$\mathbf{HI} = \mathbf{O},\tag{7}$$

where H is the $N \times J$ matrix with columns made up of the sequence of K values of h_k (see Chow *et al.* #7.6, for example) and **I** is the J-vector made up of the I_j and **O** is the N-vector made up of the O_n . The problem of obtaining the h_k is termed deconvolution, and simple and direct methods exist, however in hydrology a least-squares method has traditionally been used which we adopt here. We pre-multiply both sides of equation (7) by the transpose of H, H^T, to give

$$\mathbf{H}^{\mathrm{T}}\mathbf{H}\mathbf{I} = \mathbf{H}^{\mathrm{T}}\mathbf{O},\tag{8}$$

which is a system of J equations in J unknowns and it can be shown that this gives a least-squares solution to the problem. There are some high quality programs for the solution of such a system and we prefer to use these. However the tradition in hydrology seems to have been to premultiply by the inverse of the matrix on the left to give an explicit matrix equation for \mathbf{I} .

In this work, however, as we are addressing the gate stroking problem, we are more interested in obtaining the inverse transfer function, the one which, when convoluted with the outflow desired, gives the inflow necessary to achieve that. Hence we consider a transfer function u_k for k = 0, 1, ..., K - 1 such that we have the inverse of equation (6):

$$I_n = \sum_{j=0}^{n \le N} O_j \, u_{n-j} \,, \text{ for } n = 0, 1, \dots, N-1 \,,$$
(9)

where now the output sequence O_j has J values and the input sequence I_n has N values. Instead of equation (8) we have

$$\mathbf{U}^{\mathrm{T}}\mathbf{U}\mathbf{O} = \mathbf{U}^{\mathrm{T}}\mathbf{I},\tag{10}$$

where U is the $N \times J$ matrix where the columns are made up of the sequence of K values of u_k .

One could use the advection-diffusion approximation to obtain a transfer function for the system, however here we prefer to obtain it directly from a solution of the full St Venant equations. We use the program developed by Rubicon Systems Australia (Rubicon, 1998), which solves the equations very accurately for channel systems. As an example of the application of the program – and a demonstration of the nature of wave motion in channels, consider Figure 1, which shows the surface elevation in the first three pools of the Pyramid Hill No 1 Channel plotted each two hours for the first 24 hours. The reach is 17.6km long, and contains one siphon, whose head loss is clearly shown, plus a

small arch bridge at Dingee. A flow of 910 Ml/d at the upstream boundary was increased by 300 Ml/d but the gate at the downstream end was not moved, more clearly to demonstrate the nature of the wave motion. What happens is that a fast dynamic wave takes off and does travel the length of the channel relatively quickly, but the bulk of the motion is of a slow-moving kinematic-diffusion wave whose effect is to propagate slowly and to show a more diffusive nature.



Figure 1: Water surface profiles in the first three pools of the Pyramid Hill No 1 Channel

The conditions from that example are such as to render gate stroking not feasible, as will be shown. As an application of the method we consider a single pool of a channel with roughly the same physical characteristics as the above reach, but considerably shorter. The channel has a bottom width of 10m, batter slopes of 2:1, a slope of 0.0001, 5km long, a depth of 2m at the downstream regulator, and Manning's n = 0.025. To perform the simulation to yield the transfer function we considered a base flow of 864 Ml/d, 10 m³ / s. The inflow was increased smoothly (a Gaussian function of time) by 25% up to a maximum of 1080 Ml/d and back down to the base flow over a period of about three hours. The program then simulated conditions in the channel for a total of 20 hours.



Figure 2: Specified inflow hydrograph and computed outflow hydrograph

Results are shown in Figure 2. What is immediately obvious from the figure and which may come as some surprise is that in this relatively short pool that the "wave" has diminished by a factor of about half, and the time over which the change in discharge is felt at the outlet is considerably greater. In

fact, it is about eight or nine hours before the system has returned to steady state. Deductions based on conventional hydraulics would suggest that the wave that arrived at the outlet would be substantially the same as that which was imposed at the inlet. Interestingly, the travel speed of the peak, as measured by the time between the two peaks was about 2 m/s. According to the formula from conventional hydraulics, the speed would be about 4.4 m/s.



Figure 3: Transfer function h_k for computing outflow from inflow

We took 32 data points from the outflow hydrograph, and with J = K = 16 we used equation (8) to compute the transfer function, as shown in Figure 3, scaling the values such that the sum was equal to unity so that there would be no loss of flow. The sequence of points shown (the lines between them have no significance) corresponds to the outflow due to a single inflow spike of magnitude unity at time zero. The maximum value is about 0.35. The kind of behaviour we see in the figure seems to correspond with that observed in Figure 2, where the hump diffuses considerably and its influence is felt for a long time at the outlet.



Figure 4: Inverse transfer function u_k for computing the gate stroking solution

Then, with a view to developing a method for gate stroking we took 32 points from the inflow hydrograph and used equation (10) to compute the inverse transfer function in a least-squares sense. The results are as shown in Figure 4, where the transfer function oscillates wildly with magnitudes between -7 and +5, very much greater than for the direct transfer function. Some consideration of the mechanics, however, shows that this result might well be expected. The forward transfer function as shown in Figure 3 shows the outflow caused by a single inflow spike of magnitude unity, with the expected time shift and decay. The inverse transfer function in Figure 4, however, shows the inflow that would cause a single *outflow* spike of magnitude unity for subsequent use in gate stroking calculations. It can well be imagined, given the diffusive nature of the system as seen in the figures, that it would require a fairly remarkable input which would travel and diffuse such that all the variation combined to produce a single hump in the downstream hydrograph. One can imagine that it would require the cancellation of opposite-signed contributions in large part, as the figure suggests.

This does not mean that the solution is invalid, however. We tested it for some typical required outputs, consisting of a single smooth increase in flow from the base level, given by a tanh function. Firstly we used the function

$$\frac{Q}{Q_0} = \frac{1}{2} \left(1 + \tanh\left(1.5\left(T - 6\right)\right) \right),\tag{11}$$

where T is the time in hours. This function increases from 0 to 1 continuously, with 90% of the change of flow occurring in a period of 1.5 hours, centred at 6 hours. The results are shown in Figure 5, where the thin line shows the desired outflow (equation (11)), and the solid line shows the gate stroking solution. Huge variations of flow upstream are required by the relatively rapid change downstream. They tend to cancel each other as the wave propagates downstream so that the process of frictional diffusion ultimately leads to the single smooth increase over a short time. These are the sort of oscillations noted by many other workers in the area of gate stroking, as noted above.



Figure 5: Outflow desired and the corresponding inflow required by the gate stroking solution where 90% of the flow increase occurs within an interval of 1.5 hours

Figure 6 shows the same magnitude of increase but where it takes place over a period three times that of Figure 5 (the factor of 1.5 in the argument of the tanh function in equation (11) was reduced to 0.5). The situation for the upstream gate is now very different, for it can bring about the required downstream hydrograph with very little unnecessary motion.



Figure 6: As for Figure 5 but where 90% of the flow increase occurs within an interval of 4.5 hours

While it might be thought that this is a relatively satisfactory state of affairs, further computations soon dispel that illusion. Simply by doubling the channel length to 10km gave an inverse transfer function which oscillated even more wildly than that shown in Figure 4, varying between values of ± 40 !

These results suggest that the gate stroking problem is indeed flawed, and that it is unreasonable to require the satisfaction of precise flows at the downstream boundary by upstream gate movements. However, there is no reason not to use upstream movements to bring about a downstream hydrograph such as those shown in Figure 6, which is a reasonable approximation to a desired outflow hydrograph, such as that of Figure 5.

CONCLUSIONS

We have developed a method which can solve the gate stroking problem by techniques of linear systems theory combined with an accurate numerical simulation program, rather than having to write a suite of programs to solve the problem backwards in space and time. Use of the method shows that the problem is indeed fundamentally flawed, and that it is better to rely on traditional methods of canal operation combined with local control of the downstream regulator than to attempt to solve the gate stroking problem.

In addition the use of the method proposed has explained the various phenomena found by previous writers in this field who have attempted to implement gate stroking programs. It has shown the nature of the propagation of disturbances in channels with friction. Disturbances do not travel at the speed they are traditionally supposed to, and waves can be damped considerably and their effects smeared out in time. The application of gate stroking measures can be undertaken for channels that are not too long, but in general the precise satisfaction of the downstream requirements by upstream gate operations alone seems to be not possible. Rather, what should be used is the use of upstream gate operations to satisfy approximately the downstream requirements and the use of control measures on the downstream gate so as to achieve easily all the flows required while minimising surface disturbances.

REFERENCES

- Bautista, E., Clemmens, A. J., and Strelkoff, T. S. (1995), Inverse computational methods for openchannel flow control, In Espey, W. H. and Combs, P. G., editors, *Proc. First Int. Conf. Water Resources Engng*, San Antonio, Texas, volume 1, 593-597, New York. ASCE.
- Bodley, W. E. and Wylie, E. B. (1978), Control transients in series channel with gates, J. Hyd. Div. ASCE 104(HY10), 1395-1407.
- Burt, C. M., Gooch, R. S., Strelkoff, T. S., and Deltour, J. L. (1995), Response of ideally controlled canals to downstream withdrawals, In Espey, W. H. and Combs, P. G., editors, *Proc. First Int. Conf. Water Resources Engnng*, San Antonio, Texas, volume 1, 169-173, New York. ASCE.
- Chow, V. T. and Maidment, D. R. and Mays, L. W. (1988), *Applied Hydrology*, McGraw-Hill, New York.
- Cunge, J. A., Holly, F. M., and Verwey, A. (1980), *Practical Aspects of Computational River Hydraulics*. Pitman, London.
- Dooge, J. C. I. (1973), *Linear theory of hydrologic systems*, Technical Bulletin, 1468, United States Department of Agriculture, Washington, D.C..
- Eli, R. N., Wiggert, J. M., and Contractor, D. N. (1974), Reverse flow routing by the implicit method, *Water Resources Res.* **10**(3), 597-600.
- Falvey, H. T. (1987), Philosophy and implementation of gate stroking, In Zimbelman, D. D., editor, *Planning, operation, rehabilitation and automation of irrigation water delivery systems*, 176-179, ASCE, New York.
- Keefer, T. N. (1976), Comparison of linear systems and finite difference flow-routing techniques, *Water Resources Res.* **12**(5), 997-1006.
- Liu, F., Feyen, J., and Berlamont, J. (1992), Computation method for regulating unsteady flow in open channels, *J. Irrigation and Drainage* **118**(10), 674-689.
- Napiórkowski, J. J. (1992), Linear theory of open channel flow, In O'Kane, J. P., editor, Advances in Theoretical Hydrology A Tribute to James Dooge, chapter 1. Elsevier.
- Rubicon (1998) Draft Report on Study of SCADA Operation of Pyramid Hill No 1 Channel, Rubicon Systems Australia, Hawthorn East.
- Strelkoff, T. S., Deltour, J. L., Burt, C. M., Clemmens, A. J., and Baume, J. P. (1998), Influence of canal geometry and dynamics on controllability, *J. Irrigation and Drainage* **124**(1), 16-22.
- Szymkiewicz, R. (1993), Solution of the inverse problem for the St Venant equations, J. Hydrology 147, 105-120.
- Wylie, E. B. (1969), Control of transient free-surface flow, J. Hyd. Div., ASCE 95(1), 347-361.